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International Council for the Exploration of the Sea

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A GOAL FUNCTION OF FISHERIES

(LEGION ANALYSIS)

by

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Abstract

In Sparre 1979, a criticism of yield per recruit and F_{max} considerations, as applied by ICES working groups was presented. No alternative to the "F_{max}-method" was given in that paper. The present work is an attempt to construct an operational procedure for a rational management of international fisheries. The method is supposed to be used by bodies as e.g. the ACFM. An attempt to make a definition of what is scientific advice on fisheries management and what is political decisions is made. The Population dynamics part of the procedure is along the lines of Andersen and Ursin's model (1977) and based on the works of Helgason and Gislason (1979) and J.G.Pope (1979). The fisheries part of the model is based on Hoydal (1977) and some considerations on mixed fisheries. The rest of the procedure utilizes some basic ideas from operation research theory and some primitive economic considerations, as e.g. those presented by Gulland (1979).

This contribution is a comprehensive one, because most principal aspects of fish stock assessment are covered. I am somewhat concerned about the length of this paper, but on the other hand I feel that all the interactions between the variables of the model are of equal importance, and that it is more or less impossible to ignore some variables and make a consistent model of the remaining ones.

Contents

1. INTRODUCTION

This model deals with a number of interacting fish stocks and a number of interacting fishing fleets.
The population dynamics of fish stocks are controlled by the factors which cause the stocks to change, and they are:

Recruitment
Growth of individuals
Deaths due to fishing (including discards)
Deaths due to predation
Deaths due to "other" natural causes

Interaction between fish stocks is assumed to be caused by predation only. There is no food competition between the fish, which may cause some fish to feed at a lower rate than other fish.

Interaction between fishing fleets means that total fishing mortality on one fish stock is caused by a number of different fishing fleets.

A fleet is primarily characterized by its catch and its fishing grounds. The catch is characterized both by the species composition and the size group composition.

It is assumed that each fleet's fishery is directed against one target species. Each fleet is assumed to consist of identical vessels, as far as gear type and catching power are concerned. In the present context a fleet should be considered a management unit.

Besides the target species catch every fleet is assumed to take certain amounts of bycatches. The model attempts to take into account that "clean" fisheries are rare. Most fisheries are mixed fisheries, and consequently it is more or less impossible to make independent decisions on the effort on the various stocks. E.g. an increased effort in the cod fishery in the North Sea produces an increase of effort in the Whiting fishery.

Fishing mortalities are determined by the factors:

Gear selection (e.g. mesh size)
Fishing effort
Distribution of bycatches
Discarding
Recruitment to fishing grounds

Thus, two types of species interaction are modelled:

Biological interaction - model of predation

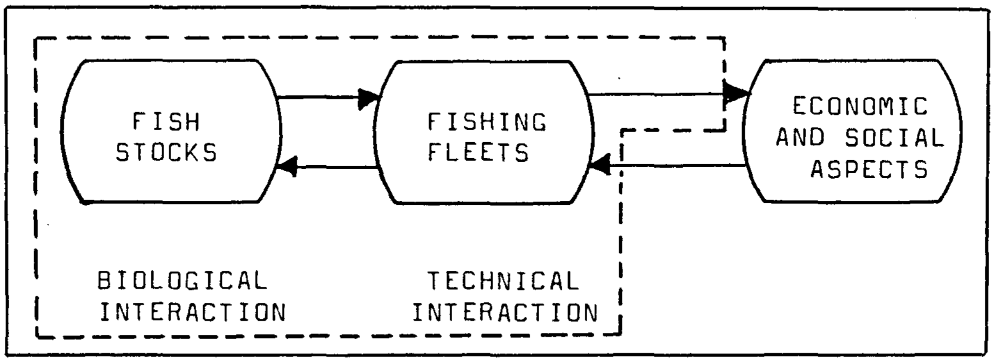
Technical interaction - model of effort distribution on fish stocks

The population dynamics of fish stocks are based on the model developed by Andersen and Ursin (1977). The present application is a reduced version of the Andersen and Ursin model developed by Pope (1979) and Helgason and Gislason(1979), the species interaction cohort analysis. A shorter name of the method is "legion analysis", (a "legion" consists of a number of "cohorts").

This work is supposed to make up the two first sections of a three section model containing :

A population dynamics sub-model
A fishing fleet sub-model
An economy and social sub-model

The connections between the three sections follow the paths shown in the figure :



The dotted line indicated the part of the total model attempted covered in the present work. It is hoped that some economy model appropriate for connection with the present model exists or will appear. Some primitive economic aspects are considered, which is indicated in the figure by the inclusion of the arrow from fleets to economy in the present model.

The model is formulated as an optimization problem. A goal function of the entire international fishery is suggested. The decision variables are :

Fishing effort
Gear (e.g. mesh size)
Bycatch

The goal is the "total value" or "total return" of the total international landings. The definition of "value" of landings is a political decision. The goal function selected for the exercise presented in this paper is to be considered as an example given for illustration purposes only.
The optimization could be subject to one (or more) constraints. These constraints are political decisions too. An example of a constraint is that certain stocks should be kept above a certain minimum level, which would prevent them from depletion.
That the problem is defined as an optimization problem does not imply that only the theoretical optimum solution should be sought. The "true optimum solution" (whatever it might be) of the fishery management problem is hoped to be somewhere in the "nearest neighbourhood" of the theoretical solution determined by aid of the present model. It would thus,

be more sensible to consider a range of solutions. In principle this method should be applied in a way similar to the traditional Y/R-curve method, i.e. return from yields obtained for a number of alternative fishing patterns should be evaluated. In fact, the present method might be considered as a generalisation of the Beverton and Holt yield per recruit method (see Appendix H).

A computer program was developed to carry out the calculations of the management procedure described in the foregoing. The program works in two steps :

STEP ONE : V.P.A. on historical data
STEP TWO : Prognosis

The program operates with a great number of options for both VPA and prognosis. One option is the traditional single species VPA and single species prognosis (e.g. as applied by the North Sea Round Fish W.G., Anon. 1980).

The predation induced interaction between fish stocks is determined from a socalled food suitability matrix. This food suitability matrix must be given as input to the program. It may be based on pure theoretical considerations on feeding behaviour of fish, but it can also be estimated from stomach content data.
The concept of food suitability is defined such that there is a one to one correspondence between the relative stomach content of predators and the food suitability matrix.
To take into account that only a certain fraction of the food consumption is met from the fish species considered in the model, it is necessary to include a compartment accounting for "other food" in the model. The treatment of the "other food"-compartment of the ecosystem is somewhat dubious, because so little is actually known about the dynamics of the invertebrates. A number of alternatives for the dynamics of "other food" will be discussed. As will appear from section 3 the concept of "other food" is important to the results of the legion analysis, especially the the predation mortality is dependent on "other food".

STEP TWO, the prognosis may be applied as either

A tactical model (short term prognosis model)

or

A strategic model (long term prognosis model)

The principal difference between these two applications of STEP TWO, is that the strategic model must include a stock/recruitment model.
The tactical model is used for TAC calculations, i.e. a prognosis for only two years.

The year class strength is usually known for first year from young fish survey data. For the next year (the year for which the TAC is calculated) year class strength is usually of little importance.

The strategic model is supposed to predict the development for a period of, say, 5-25 years. Most likely, the fishing patterns will be assumed to remain constant from year to year and we wish to run the strategic prognosis for as many years as the system needs to arrive at a stable situation. The strategic model will usually be used to make desicions about what the general trends in effort of future years should be in order to optimize the long term yield from fisheries.
For a prognosis of more than, say, 5 years, the stock/recruitment is one of the main factors determining the dynamics of the system.

The data requirement of the present model is higher than for the traditional assessment models.

All data necessary for the single species assessment is also needed for the legion analysis. In addition to that, data for the estimation of the food suitability matrix should be collected.

Thus, the problem, usually met in ICES WGs, caused by incomplete data bases, is not solved in this work. On the contrary, application of the multispecies-multi fleet model will throw light on the gaps in the data base used in current assessment.

This contribution may be said to raise more problems that it solves. If ICES accept to apply models along the lines suggested in this paper, the conclusion may well be that ICES WGs are unable to make proper scientific assessments, unless the current level of data collection is considerably increased. As the first step this work is supposed to be used in a discussion of what data base is actually needed for an ICES WG to make an assessment. To assess the importance of the different parameters trial runs of the model with a range of guessed values of those parameters which can not be estimated today, should be made.

For a discussion of the current setting of TACs, see Macer, Jones and Bannister, 1979.

Thus, I suggest that ICES as soon as possible start to use more realistic models (Cf. App. H), but I doubt the advisability of suggesting that they should replace the traditional methods in the setting of TACs.

In my opinion TACs should only be given for those stocks which are obviously threatened (such as the herring) as long as the current data base is imcomplete.

2. LIST OF SYMBOLS

Below is a complete list of symbols applied in this paper. Due to notational convenience symbols slightly different from the commonly used ones are applied. When convenient the symbol is given a definition in this section, otherwise reference to the section containing the proper definition is given.

a	: index of agegroup
b	: index of agegroup
B(y,s,a)	: Biomass at the beginning of year y (=N(y,s,a) $\bar{w}$ (s,a))
BYC(e,s)	: Bycatch matrix (see section 4.2)
C(y,s,a)	: number caught during year Y (= number landed + number discarded)
d	: index of agegroup
D(y,s,a)	: number of deaths due to predation during year y
DISC(e,s)	: term in the expression for discards (see section 4.1)
e	: index of fleet
E	: total number of fleets, e = 1,2,...., E.
EF(e,y)	: fishing mortality on the target species of fleet e subject to maximum exploitation (see section 4.1)

EGG(y,s)	: total number of hatching larvae (see section 5.2)
$\overline{\text{FLAND}}$(e,y,s,L)	: Landing (fishing) mortality on target species as a function of length exerted by fleet e (see section 4.1)
$\overline{\text{FDISC}}$(e,y,s,L)	: Discard mortality on target species as a function of length exerted by fleet e (see section 4.1)
$\overline{\text{F}}$(e,y,s,L)	: Total fishing mortality on target species as a function of length exerted by fleet e
FLAND(e,y,s,a)	: Landing (fishing) mortality on target species as a function of age exerted by fleet e (see section 4.1)
FDISC(e,y,s,a)	: Discard mortality on target species as a function of age exerted by fleet e (see section 4.1)
F(e,y,s,a)	: total fishing mortality on target species as a function of age exerted by fleet e (see section 4.1)
FLAND(y,s,a)	: $\sum_e$ FLAND(e,y,s,a) where e is index of fleet (see section 4.2)
FDISC(y,s,a)	: $\sum_e$ FDISC(e,y,s,a) where e is index of fleet (see section 4.2)
F(y,s,a)	: $\sum_e$ F(e,y,s,a) where e is index of fleet (see section 3.1 and 4.2)
FBLAND(e,y,s,a)	: as FLAND, but for bycatch species s (see section 4.2)
FBDISC(e,y,s,a)	: as FDISC, but for bycatch species s (see section 4.2)
FBYC(e,y,s,a)	: as F, but for bycatch species s (see section 4.2)
FBLAND(y,s,a)	: as FLAND, but for bycatch species (see section 4.2)
FBDISC(y,s,a)	: as FDISC, but for bycatch species s (see section 4.2)
FBYC(y,s,a)	: as F, but for bycatch species s (see section 4.2)
$\underline{\text{F}}$	: vector of fishing mortalities (see section 6)
FOOD(s,a)	: Total food consumption per individual per year
GSEL(s,e,L)	: term in the expression for gear selection curve (see section 4.1)
i	: index of species
j	: index of species
k	: index of time period during the first year of life (see section 3.4)
K(s)	: von Bertalanffy parameter (see "LENGTH")
K(y)	: capital (see section 6)
L	: individual length (as independent variable)
LENGTH(s,t)	: length at age t: L8(s)(1-exp(-k(s)(t-to(s)))).
L(s,a)	: average length of age group a: LENGTH(s,a+.5)
L50%(s,e)	: the length at which 50% of the fish entering the gear of fleet e is retained in the gear(see section 4.1)
L75%(s,e)	: as L50%(s,e) (see section 4.1)
LL(s,e)	: L75%(s,e)/L50%(s,e)(see section 4.1)
LD50%(s,e)	: the length at which 50% of the fish caught are not discarded (see section 4.1)
LD75%(s,e)	: as DL50% (see section 4.1)
M1(s,a)	: residual natural mortality (not predation induced natural mortality)
M2(y,s,a)	: predation induced natural mortality (see section 3.1 or appendix B)
M20(y,s,k)	: predation induced natural mortality in the first year of life (see section 3.4)
MAXEF(y,e)	: maximum effort of fleet e (see section 6)
MINEF(y,e)	: lower limit of fleet e's effort (see section 6)
MINSSB(s)	: minimum allowabel spawning stock biomass (se section 6)
MESH(e)	: Mesh size (or a gear parameter corresponding to mesh size (see section 4.1)

MAGE(s)	: first age of maturity.
N(y,s,a)	: stock number at the beginning of year y
$\overline{N}$(y,s,a)	: average stock number during year y: N(y,s,a)(1-exp(-Z(y,s,a))/Z(y,s,a)
NO(y,s,k)	: stock number at the beginning of period k in the first year of life (see section 3.4)
$\overline{NO}$(y,s,k)	: average stock number during period no k in the first year of life NO(y,s,k)(1-exp(-ZO(y,s,k)T(k)))/ZO(y,s,k)T(k) see section 3.4)
NOMAX(s)	; Maximum number of recruits (see section 5.2)
OAGE(s)	: oldest agegroup
OF(s,j,b,k)	: term in the expression for M2O (see section 3.4)
OGROUP(y,j,b)	: biomass of o-group food fish available to predator j agegroup b (see section 3.4)
OTHER FOOD	: The biomass of the ecosystem considered is partitioned into two: Biomass of "considered" fish species Biomass of "other" animals In the present context "considered fish" is simply the S named fish species considered in the model. Usually considered fish species will be the same as "commercially important species". Other animals account for all other fish species and invertebrates, which may occur as prey for any of the considered fish species. Biomass of other animals is designated "OTHER FCOD" to emphasize that it is as prey for considered fish species that the concept of "other animals" is important to the present model. Other food is to be considered as a homogeneous mass of food available to all considered fish species. This rather artificial concept is introduced only in order to reduce the mathematical complexity of the model.
q(s)	: condition factor
r	: rate of interest (see section 6)
REC(s,e,L)	: term in the expression for recruitment to fishery (see section 4.1)
RECL50%(s,e)	: the length at which 50% of the fish are recruited to the exploited part of the stock (see section 4.1)
RECL75%(s,e)	: as RECL 50% (see section 4.1)
RGSEL(s,e,L)	: term in the expression for the right hand side slope of the gear selection curve (see section 4.1)
RL50%(s,e)	: L50% for the right hand slope of the gear selection curve (see section 4.1)
RL75%(s,e)	; as RL50%(s,e) (see section 4.1)
RETURN	: return from fisheries (see section 6)
s	: index of species
S	: number of considered fish species. s = 1,2,...,S.
SEL(s,e)	: selection factor
STOC(s,a,j,b)	: relative stomach content (see section 3.3)
SPAW(s,a)	: number of hatching larvae per kg spawning stock (see section 5.2)
SSB(y,s)	: spawning stock biomass at the beginning of year y: $\sum_{a \geq MAGE(s)} B(y,s,a)$

SUIT(s,a,j,b)	: food suitability. SUIT is a measure of the suitability of prey species s (age group a) as food for predator species j (age group b). One possibility is to define SUIT as Pope (1979) does. Another possibility is given by Andersen and Ursin (1977) and applied by Helgason and Gislason (1979) and Anon (1980). In section 3.3 it is demonstrated how SUIT can be determined on a purely emperical basis, i.e. how SUIT can be estimated from stomach content samples.
t	: time
t0	: von Bertalanffy parameter (see "LENGTH")
T(k)	: length of time period in the first year of life (see section 3.4)
TOTB(y)	: total biomass of the ecosystem at the beginning of year y: $\sum_s\sum_a B(y,s,a)$+OTHER FOOD
V(y,e,s,a)	: return-value of landings (see section 6)
$\overline{w}$(s,a)	: average body weight
wo(s,k)	: average body weight in period k in the first year of life
y	: index of year
YFIRST	: first year considered using historical data
YLAST	: last year for which catches are konwn
YFOR	: last year for which prognosis is made
YAGE(s)	: youngest age group
YIELD(y,e,s,a)	: yield of fleet e (see section 6)
Y(y,s,a)	: yield from species s: $\sum_e$ YIELD(y,e,s,a) (see section 6)
Z(y,s,a)	: total mortality: M1(s,a)+M2(y,s,a)+F(y,s,a)
Z0(y,s,k)	; total mortality in period k in the first year of life (see section 3.4)

3. POPULATION DYNAMICS.

The population dynamics model is based on J. Pope (1979) and Helgason & Gislason (1979).

Independent of each other these two parallel works were developed at the same time. There are some differences in the two models, but the basic principles are the same, namely the way ordinary VPA is extended to include predation induced species interaction.

The model will be referred to in the following as "legion analysis". Legion analysis may be considered as a time discrete reduced version of the Andersen and Ursin model (1977).

At its meeting in March 1980, the ICES Ad. hoc. WG. on multispecies assessment model testing recommended that an international stomach sampling program should be implemented in the North Sea in 1981 (Anon. 1980). The theoretical basis for this investigation is the legion analysis.

The population dynamics part of this paper may be considered as my suggestion to how the observations from the planned stomach sampling in 1981 can be incorporated into the ICES assessment of North Sea stocks.

3.1 SPECIES INTERACTION COHORT ANALYSIS (LEGION ANALYSIS)

There are three basic equations in legion analysis. The two of them are those of ordinary single species VPA:

$$N(y+1,s,a+1)=N(y,s,s)\exp(-Z(y,s,a)) \qquad (3.1)$$

$$C(y,s,a)=F(y,s,a)N(y,s,a) \qquad (3.2)$$

(Recall: $\bar{N}(y,s,a)=N(y,s,a)(1-\exp(-Z(y,s,a)))/Z(y,s,a)$).

The new thing in legion analysis compared to ordinary VPA is the partitioning of Z into three parts

$$Z=F+M1+M2$$

M1 plays the same role in legion analysis as M(=M1+M2) in ordinary single species VPA. D(y,s,a), the number of deaths due to predation is calculated by an equation similar to that for the catch:

$$D(y,s,a)=M2(y,s,a)\bar{N}(y,s,a) \qquad (3.3)$$

The three equations 3.1-3 define the multispecies cohort analysis developed by Pope (1979) and Helgason & Gislason (1979). (For a detailed explanation see the original sources or Appendix B.)

M1 is an exogenous parameter and M2 is calculated by:

$$M2(y,s,a)=\sum\sum FOOD(j,b)N(y,j,b)\frac{SUIT(s,a,j,b)}{\sum_i\sum_d \bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d)} \qquad (3.4)$$

By putting all SUIT(s,a,j,b)=0 all M2(y,s,a) become zero (see Eq.3.4), and the legion analysis reduces to a number of independent ordinary singlespecies VPAs. The theoretical definition of the food suitability matrix SUIT will not be discussed. This does not mean that the definition of SUIT is considered an unimportant detail, but rather that I prefer to let it depend on the conclusions to be drawn from the stomach content sampling scheme in 1981 (Anon. 1980). In Anon. 1980 the definition of SUIT given in Andersen & Ursin (1977) was adopted. However, I feel that this definition should only be considered as a preliminary one. In section 3.4 an attempt is made to relate SUIT to stomach content data.

3.2 FOUR ALTERNATIVE ASSUMPTIONS FOR THE TREATMENT OF "OTHER FOOD"

In the legion analysis developed by Helgason and Gislason the fraction of total food met from the considered fish species is not assumed to remain constant. Pope assumes this fraction (Y in his notation) to remain constant, which to my opinion makes Pope's model inconsistent. This is why I adopt the idea of Helgason and Gislason and introduce the concept of "Other food". What goes wrong in Pope's model is that predation mortality becomes approximately inversely proportional to stock size of prey. As a simple illustration, let us consider a system containing only cod and herring. According to Pope, a constant percentage, say 20%, of cod's food is always herring. If the cod stock remains constant and the herring stock decreases, the percentage of the herring stock eaten by cod increases. By introducing "other food" this mechanism can be avoided, since cod then will switch to "other food" as the herring stock declines, and predation mortality on herring will remain nearly constant. Generally speaking, predation mortality should be proportional to the density of predators, but independent of prey density, exactly as fishing mortality is proportional to fishing effort.

In the present version of legion analysis the fraction met from fish species included in the VPA is simply

$$U = \frac{\text{available biomass of prey fish}}{\text{total available biomass of food}}$$

That is: S is the number of considered fish species, and the number of "other food" animals is designated N(y,S+1,a). "Other food" is assumed to contain one agegroup only, and the weight of one specimen of "other food" is arbitrarily put equal to one. Thus the available biomass of other food is $\bar{N}(y,S+1,1)\cdot SUIT(S+1,1,j,b)$ for predator j age group b and

$$U = \frac{\sum_{i=1}^{S} \sum_{d} \bar{N}(y,i,d) SUIT(i,d,j,b)\ \bar{w}(i,d)}{\sum_{i=1}^{S+1} \sum_{d} \bar{N}(y,i,d) SUIT(i,d,j,b)\ \bar{w}(i,d)}$$

The total biomass of the ecosystem TOTB is assumed to remain constant in this version of legion analysis

$$TOTB(y) = \sum_{i}^{S+L} \sum_{d} \bar{N}(y,i,d)\ \bar{w}(i,d) = \text{constant}$$

The available biomass of other food, thus becomes:

$$\left(TOTB - \sum_{i=1}^{S} \sum_{d} \bar{N}(y,i,d)\ \bar{w}(i,d)\right) SUIT(S+1,1,j,b)$$

and the totaæ biomass of food available to predator j,b may be written:

$$\sum_{i=1}^{S} \sum_{d} N(y,i,d) w(i,d) (SUIT(i,d,j,b) - SUIT(S+1,1,j,b)) + TOTB\cdot SUIT(S+1,1,j,b)$$

which demonstrates that available biomass of food may vary from year to year.

In the model of Helgason and Gislason the biomass of OTHER FOOD is assumed to remain constant from year to year whereas total biomass of the ecosystem may vary.

Another possibility is to assume the total available biomass of food for every predator to remain constant (Ursin, personal communication). This assumption follows naturally from the assumption of constant feeding rate.

The assumption made by Pope may be formulated as the assumption: OTHER FOOD = 0, which should be interpreted as an ignoring of OTHER FOOD, and consequently feeding rate should be given a lower value in the Pope model that in the other models.

To assess the principal differences between these four models, we shall consider M2(y,s,a) as a function of prey abundance $\bar{N}(y,s,a)$. The consumption $\bar{N}(y,j,b)FOOD(j,b)$ by predator (j,b) is in this context assumed to remain constant. The four models give:

<u>Pope</u>: $$M2(y,s,a) = \sum_{j=1}^{S} \sum_{b} \frac{\bar{N}(y,j,b) FOOD(j,b) SUIT(s,a,j,b)\cdot \text{constant}}{\sum_{i=1}^{S} \sum_{d} \bar{N}(y,i,d) SUIT(i,d,j,b) \bar{w}(i,d)}$$

where: $$\text{constant} = \frac{\text{consumption met from fish considered}}{\text{total consumption}}$$

<u>Helgason and Gislason</u>:

$$M2(y,s,a) = \sum_{j=1}^{S} \sum_{b} \frac{\bar{N}(y,j,b)FOOD(j,b)SUIT(s,a,j,b)}{\sum_{i=1}^{S} \sum_{d} \bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d) + constant}$$

where: constant = $\bar{N}(y,S+1,1)\bar{w}(S+1,1)SUIT(S+1,1,j,b)$

<u>Ursin</u>: $$M2(y,s,a) = \sum_{j=1}^{S} \sum_{b} \frac{\bar{N}(y,j,b)FOOD(j,b)SUIT(s,a,j,b)}{constant}$$

where : constant = $\sum_{i=1}^{S+1} \sum_{d} \bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d)$

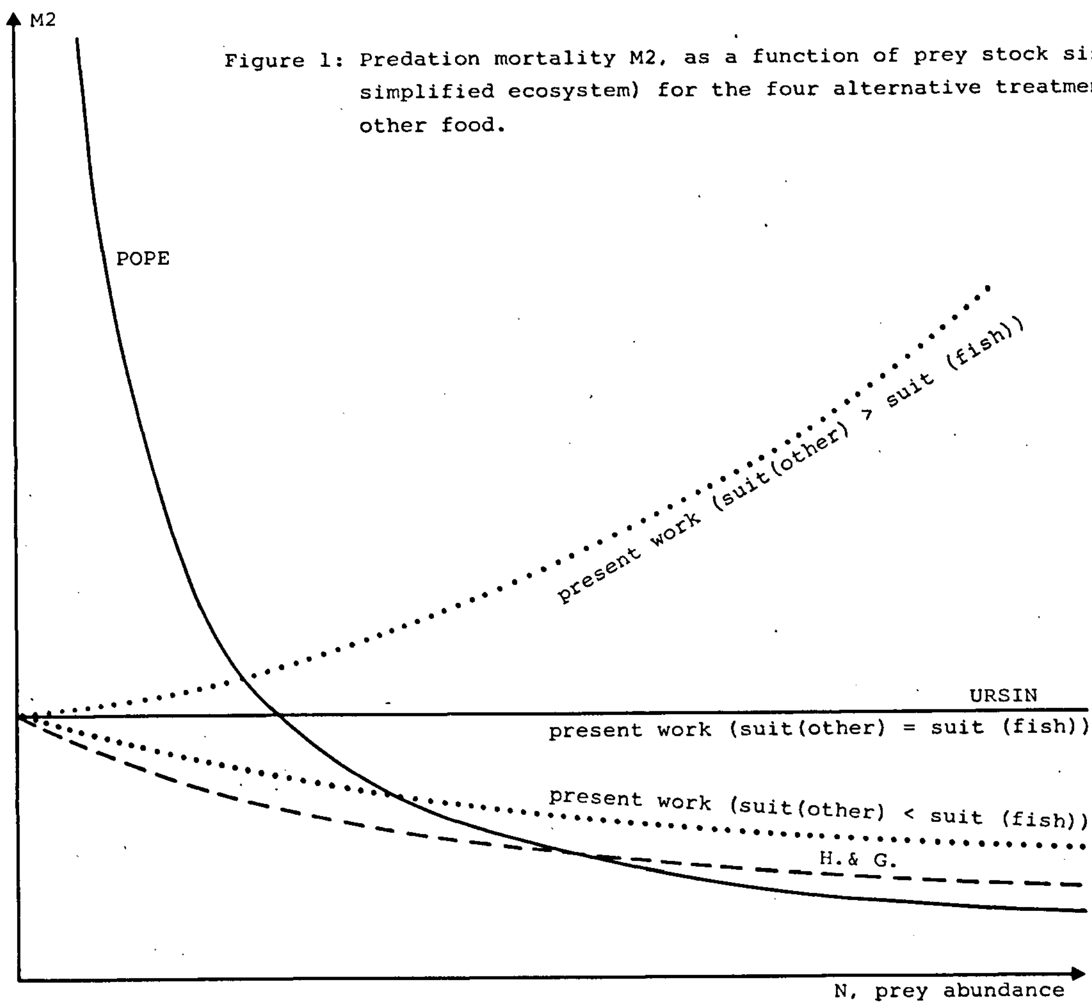


Figure 1: Predation mortality M2, as a function of prey stock size (in a simplified ecosystem) for the four alternative treatments of other food.

present work:

$$M2(y,s,a)=\sum_{j=1}^{S}\sum_{b}\frac{N(y,j,b)FOOD(j,b)SUIT(s,a,j,b)}{\sum_{i=1}\sum_{d}\bar{N}(y,i,d)\bar{w}(i,d)(SUIT(i,d,j,b)-SUIT(S+1,1,j,b))+constant}$$

where: $constant = \sum_{i=1}^{S+1}\sum_{d} \bar{N}(y,i,d)\bar{w}(i,d)SUIT(S+1,1,j,b)$

$= TOTB(Y)SUIT(S+1,1,j,b)$

If we consider a simple system consisting of one predator and one fish prey and other food, all represented by a single agegroup the principal features of the four models become clearer. Figure 1 shows M2 as a function of prey abundance in such a simple model. As appear from the formula, M2 as defined in the present work depends on the ratio between SUIT for the fish prey and for other food.

3.3 ESTIMATION OF FOOD SUITABILITY MATRIX FROM STOMACH CONTENT DATA

Total consumption of predator j age group b is

$\bar{N}(y,j,b)\ FOOD(j,b)$

The consumption of prey species s age group a is

$$\bar{N}(y,j,b)\ FOOD\ (j,b)\frac{\bar{N}(y,s,a)\bar{w}(s,a)SUIT(s,a,j,b)}{\sum_{i}\sum_{d}\bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d)}\ .$$

$\sum_{i}\sum_{d}\ \bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d)$

is the biomass of food available to predator j age group b.

$\bar{N}(y,s,a)\bar{w}(s,a)SUIT(s,a,j,b)$

is the available amount of prey species s age group a to the predator.

$$STOC(s,a,j,b) = \frac{\bar{N}(y,s,a)\bar{w}(s,a)SUIT(s,a,j,b)}{\sum_{i}\sum_{d}\bar{N}(y,i,d)\bar{w}(i,d)SUIT(i,d,j,b)}$$

defines for fixed j,b and variable s,a the relative stomach content of predator j age group b. Notice that

$$\sum_{s}\sum_{a}\ STOC(s,a,j,b) = 1.0$$

STOC(s,a,j,b) is the theoretical relative stomach content calculated within the model. It is determined from SUIT and the biomass $\bar{N}\bar{w}$. On the other hand, STOC can also be estimated from stomach content investigations, which would provide us with a test of the assumptions made about SUIT. But also SUIT could be estimated directly from stomach contents survey data. i.e. the values of SUIT(s,a,j,b) could be calculated from the observed values of STOC (s,a,j,b). To establish such a one to one correspondance between SUIT and STOC we got to put an extra constraint on SUIT(due to pure mathematical regards). If one looks at formula 3.4 it appears that a multiplication of all SUIT's by the same constant would not change Eq. 3.4. That is, without reducing the biological properties of SUIT, we can add the constraint

$$\sum_{s}\sum_{a} \mathrm{SUIT}(s,a,j,b) = 1.0$$

to the definition of SUIT. A series of algebraic manipulations applied to formula (9) shows that

$$\mathrm{SUIT}(s,a,j,b) = \frac{\left(\dfrac{\mathrm{STOC}(s,a,j,b)}{\bar{N}(y,s,a)\bar{w}(s,a)}\right)}{\sum\limits_{i}\sum\limits_{d}\dfrac{\mathrm{STOC}\ (i,d,j,b)}{\bar{N}(y,i,d)\bar{w}(i,d)}} \qquad (3.5)$$

For a detailed derivation of Eq.3.5 see Appendix F .

So if stomach content data are available by prey species and age group for all predator species considered, and legion analysis output is done, one actually does not need bother about the definition of SUIT. The intricate aspect is that we need to know SUIT before a legion analysis can be carried out, but for the moment we shall forget this and postpone the discussion to the end of the section.

SUIT is assumed to remain constant from year to year. That is, we assume the feeding behaviour to remain unchanged, if available food remains constant. Thus, SUIT could be estimated as the average value for a series of years. In Appendix C a hypothetical example of the calculation of SUIT from stomach content data and legion analysis output is given. Stomach investigations applicable to the present purpose should contain:

I: Ageing of predators
II: Species determination and ageing of prey

Ageing may be carried out by length measurements and conversion to age by an age/length key. The minimum demand to the prey specification is that stomach contents are separated into all considered species and age groups and other food. Table 2 in Appendix C shows the minimum type of information necassary for the present assessment (for a detailed discussion see Amon. 1980).

The problem of how SUIT can be calculated when $\bar{N}$ is unknown may be solved by means of the following iterative procedure:

1. Make an initial guess on SUIT
2. Estimate $\bar{N}$ (by legion analysis)
3. Estimate SUIT. If two successive estimates of $\bar{N}$ and SUIT deviate more than a certain maximum allowed deviation, then go to 2.

One doubtful aspect of this approach is that the first time the method is likely to be used, is the year after the stomach content survey has been carried out. For that year the fishing mortalities are usually badly estimated (some times even guessed), which results in poor estimates of stock numbers.

Thus, to estimate food suitability coefficients it is necessary to have precise information on fishing effort (to obtain good estimates of fishing mortalities of the final year) so that stock sizes can be estimated with an acceptable precision.

3.4 PREDATION IN THE EARLY LIFE OF FISH

In the foregoing it was discussed how the suitability matrix could be estimated from stomach content observations (STOC), stocknumbers ($\bar{N}$) and body weights $\bar{w}(s,a)$, by Eq.(3.5). No detailed description of how the average body weight should be estimated was given in that section. For the fish older than 1 year the definition of $\bar{w}$ could be average annual weight. This concept could be given a proper mathematical definition, but for the present purpose the intuitive concept should be sufficient.

However, we may run into problems with the consistency of the model if it turns out that the average body weight of prey in the sea differs from that found in the stomachs of predators. For the larger prey (1 year old or older the latter source of error is assumed to be negligible.

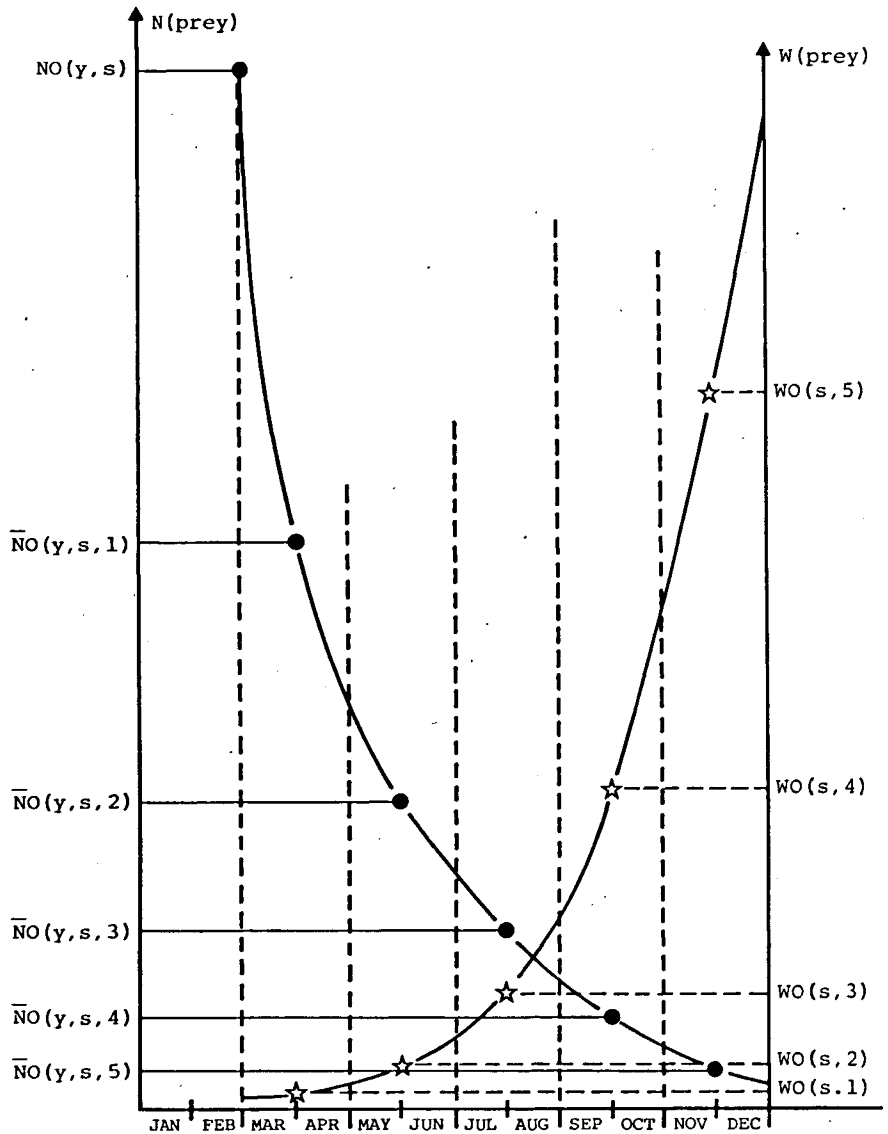
For the 0-group the definition of $\bar{w}(s,a)$ is more problematic. From birthday to the first of January next year the young fish may have increased their weight by a factor ranging from 1000 to 10000,and the stock number may have been reduced by a factor from .0001 to .01 (depending on the definition of "birthday"). Thus, it is not obvious which values for $\bar{N}$ and $\bar{w}$ to apply for the 0-group prey.

The species interaction VPA and prognosis can operate for the 1-group and older fish exclusively, by considering the 0-group on Jan. 1. as the recruits (i.e. the new 1-group). But as predation mortality is supposed to act as an important stock reducing factor in the first year of life it would be disadvantageous to exclude the early stages from an exercise which focuses on predation mortality. Further, it is hoped that a part of the stock recruitment relationship may be approached by considering the predation mortality in the first year of life.
The approach to be suggested now, is what to my opinion is the simplest one which take into account observed facts from stomach content investigations.

Figure 2 shows the dynamics of a 0-group from its birthday to Jan. 1. next year. This period is partitioned into a number of shorter periods. In the present (hypothetical) example there are five periods each of duration two months. From weight at age a partitioning of time is transferred into a grouping of body weights as shown in Fig. 3.

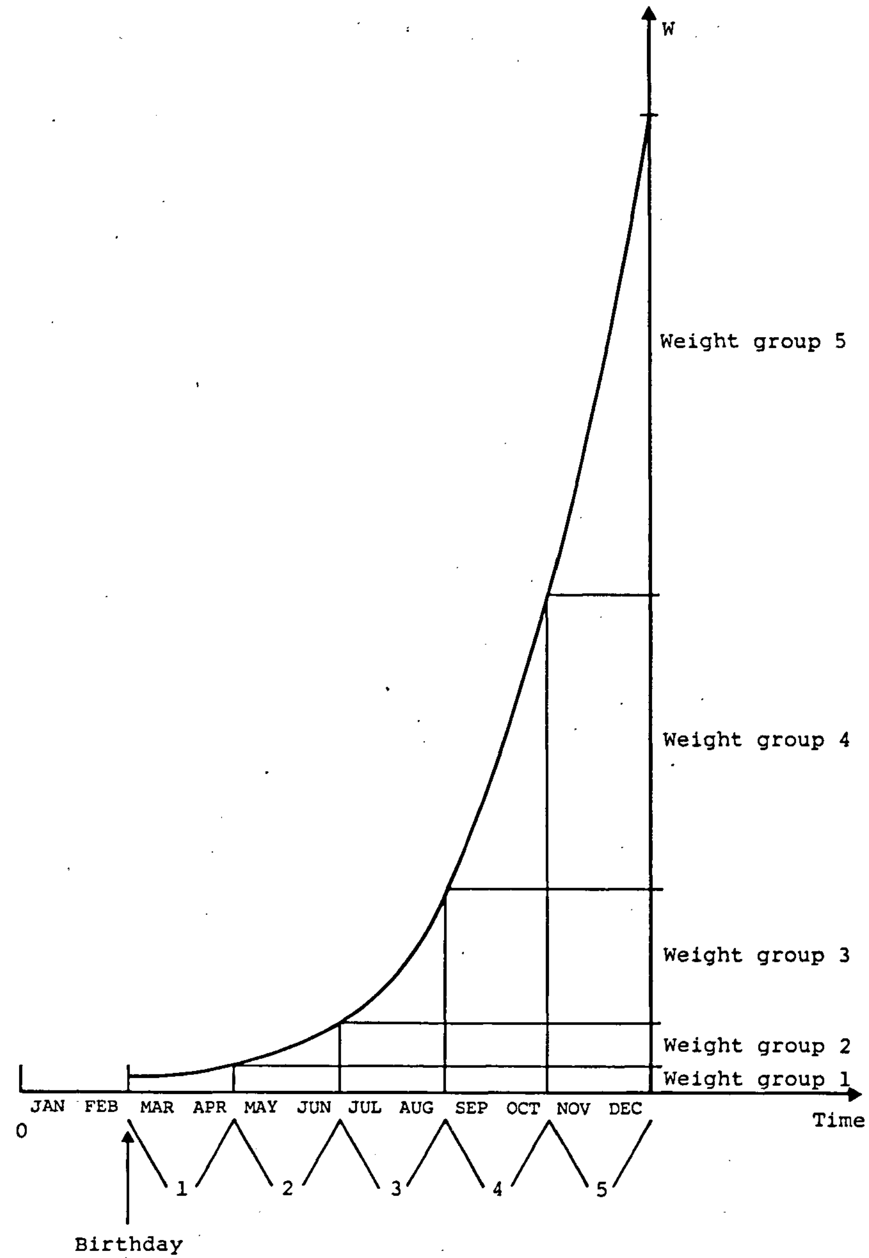
From stomach content samples the mean weights of prey in the stomach of all predators (species and age groups) are assumed to be known.

Figure 2: Definition of predation pattern in the first year of life.



Period	1	2	3	4	5
Cod	—	—	1	2	3-14
Whiting	—	1	2	3	4-10
Saithe	—	—	1	2-3	4-14

Figure 3: Definition of weight groups in the first year of life.



To each predator corresponds one of the five weight groups of Fig. 3, namely the weight group to which the average weight of the prey in its stomach belongs.

The first approximation in this approach is to assume that each predator only eats 0-group prey species s from one of the five weight groups.

This implies e.g. that if stomach investigations of two years old whiting show that on average they eat 0-group prey from weight group 3 (see Fig. 3) they are only allowed to eat 0-group species s during period 3 (July and August). The rest of the year the two year old whiting is assumed not to eat any 0-group fish of species s.

It is further assumed that all 0-group eaten from a particular weight group have the same weight, namely

wO(s,k) = the average weight of 0-group fish species s weight group k.

In the bottom of Fig. 2 a (hypothetical) example of allocation of prey weight groups on predators is given. Notice that each age group of a predator only occurs once in the table.

Let ZO(y,s,k) be the total mortality in period k. Let NO(y,s,k) be the stock number at the beginning of period k. The average stock number in period k is

$$\overline{NO}(y,s,k) = NO(y,s,k)(1-\exp(ZO(y,s,k)T(k)))/ZO(y,s,k)T(k)$$

where T(k) is the duration (years) of the time period k.

As NO(y,s,k) = NO(y,s,k+1)exp(Z(y,s,k)T(k)),this equation may also be written in the "backwards" version:

$$\overline{NO}(y,s,k) = NO(y,s,k+1)(\exp(ZO(y,s,k)T(k))-1)/ZO(y,s,k)T(k) \qquad (3.6)$$

Let M2O(y,s,k) be the predation mortality of species s agegroup 0 in time period k. Then we define

$$M2O(y,s,k) = \sum_j \sum_b FOOD(j,b)\ \overline{N}(y,j,b) \cdot \qquad (3.7)$$

$$\cdot \frac{SUIT(s,0,j,b)\ OF(k,s,j,b,k)}{\sum_i \sum_d \overline{N}(y,i,d)SUIT(i,d,j,b)\overline{w}(i,d) + \sum_i \overline{NO}(y,i,k)SUIT(i,0,j,b)OF(i,j,b)\overline{wo}(i,k)}$$

$$\text{where } OF(s,j,b,k) = \begin{cases} 1 & \text{if 0-group s is eaten by (j,b) in period k} \\ 0 & \text{otherwise} \end{cases}$$

For the estimation of M2 for agegroups a $>$ 1 the expression

$$M2(y,s,a) = \sum_j \sum_b FOOD(j,b)N(y,j,b) \cdot \frac{SUIT((s,a,j,b)}{\sum_i \sum_{\substack{d \\ d>0}} \bar{N}(y,i,d)SUIT(i,d,j,b)\bar{w}(i,d) + OGROUP(y,j,b)} \qquad (3.8)$$

where

$$OGROUP\ (y,j,b) = \sum_k \sum_i \overline{NO}(y,i,k)SUIT(i,0,j,b)OF(i,j,b,k)\overline{wo}(i,k)$$

is suggested.

The backwards VPA calculation on historical data, in which the 0-groups are treated as described above becomes:

A: Make a guess on the available biomass of 0-groups (i.e."OGROUP" in Eq. 3.8)

B: Perform a legion analysis on all agegroups older than 0 years. Notice that biomass of 0-groups is included in the calculation of available food, but yet, no 0-group fish are eaten.

C: If the VPA results of the current iteration is equal to those of the previous iteration, then go to FINIS;
k:=5; (k is index of time period for the 0-groups).

D: Make a guess on Z0(y,s,k);

E: Calculate $\overline{N0}$(y,s,k); (Eq.3.6)
Calculate M20(y,s,k); (Eq.3.7) (At this stage of the calculations the 0-groups are devoured)
Z0: = M10 + M20;
If Z0 of the current (local) iteration is different from that one found in the previous (local) iteration, then go to E;

F: k: = k-1; if k$>$ 0 then go to D;

G: Calculate available biomass of 0-groups as prey for each predator; go to B;

FINIS

To take fishing on the 0-groups into account requires that the catches of 0-groups are given for each of the weight groups(defined by Fig. 3). The calculation of fishing mortalities for each time period of the first year of life ,F0(y,s,k), is performed as the calculation for the older agegroups.

To include predation of 0-groups on 0-groups is possible. It may be important to include the interaction between the juveniles (cf. Robb, A.P. and Hislop, J.R.G. 1980). The summation over predators in Eq. (3.7) (index j,b) may well be extended to include the 0-groups as predators. To let the 0-groups eat older fish would require drastic extensions of the model and computer time required. E.g. it is not possible to let 0-group cod eat 1-group sprat, in the present version of the model.

4. FISHING FLEET MODEL

The fishing fleet model to be described in this section only applies to the prognosis part of the model.

Each fleet is assigned a target species. Several fleets could have the same target species, but a fleet can only have one target species. The idea behind the concept of "target species" is that TACs (and most other limitations on fishery) only act as regulating factors on the fishery on those stocks at which the fisheries are directed. One consequence of this is that TACs for the various stocks should not be given independently of each other. For example when setting a TAC on North Sea whiting it should be taken into account that whiting is primary taken as by-catch in the cod fishery. So if e.g. the cod quota is high and the whiting quota is low we may well end up in a situation where saleable whiting must be discarded if the cod quota should be taken. To avoid such unnecessary losses, the quotas should be adjusted to each other.

4.1 FISHING MORTALITY ON TARGET SPECIES

The following symbols are used:

SEL(s,e) : selection factor for (target) species s, being caught by fleet s
MESH(e) : mesh size (cm) used by fleet e (or a parameter corresonding to mesh size)
L50%(s,e): SEL(s,e) MESH(e) = the length of(target) species s, at which 50% of the fish entering the gear of fleet e is retained in the gear
L75%(s,e): defined as L50%
LL(s,e) : L75%(s,e)/L50%(s,e)
EF(e,y) : Maximum (subject to length) fishing mortality on the target species of fleet e in year y

The fishing mortality exerted by fleet e on target species s of length L is defined as follows:

$$\bar{F}(e,y,s,L) = EF(e,y)GSEL(s,e,L)/(GSEL(s,e,L)+1) \qquad (4.1)$$

$$GSEL(s,e,L) = \exp\left(\frac{L - L50\%(s,e)}{L75\%(s,e) - L50\%(s,e)} \log 3 \right)$$

For a detailed explanation of this formula see Appendix G , Hoydal, 1977 or Hoydal et. al., 1980.

A fraction of F is discard mortality. This fraction is

1-DISC(s,e,L)/(1+DISC(s,e,L)) = 1/(1+DISC(s,e,L)) where

$$DISC(s,e,L) = \exp\left(\frac{L - LD50\%(s,e)}{LD75\%(s,e) - LD50\%(s,e)} \log 3 \right)$$

Thus, discard mortality is

$\bar{F}(e,y,s,L)/(1+DISC(s,e,L) = \overline{FDISC}(e,y,s,L)$

and landing mortality is

$F(e,y,s,L)DISC(s,e,L)/(1+DISC(s,e,L) = \overline{FLAND}(e,y,s,L)$

Fishing mortality on each age group is assumed to remain constant during a year. Average length of a one year old fish is L(s,a) and fishing mortality on target species s agegroup a exerted by fleet e in year y is

$$F(e,y,s,a) = \bar{F}(e,y,s,L(s,a))$$
$$FLAND(e,y,s,a) = \overline{FLAND}(e,y,s,L(s,a))$$
$$FDISC(e,y,s,a) = \overline{FDISC}(e,y,s,L(s,a))$$

(see Figure 4)

The gear selection curve may also have a descending slope in the right-hand side. I.e. if larger fish are assumed to have less probability of being caught than medium sized fish, the curve may have a form as shown in Figure 5.

A curve of this shape can be obtained by multiplying $\bar{F}$ of formula (4.1) by a factor 1/(1+RGSEL) where

$$RGSEL(s,e,L) = \exp\left(\frac{L-RL50\%(s,e)}{RL75\%(s,e)-RL50\%(s,e)}\log 3\right)$$

Thus Eq. (4.1) becomes

$$\bar{F}(e,y,s,L) = EF(e,y)\cdot\frac{GSEL(s,e,L)}{1+GSEL(s,e,L)}\cdot\frac{1}{1+RGSEL(s,e,L)} \qquad (4.2)$$

The young fish may not be fully recruited to the fishing grounds at the age (or length) where fishing on them starts. To take this into consideration a third factor should be multiplied to the expression in Eq.(4.2). This factor should be the fraction of the stock recruited to the fishing grounds at a given length.

The factor can be defined as the other selective factors:

$$\frac{REC(s,e,L)}{1+REC(s,e,L)} \qquad \text{where}$$

$$REC(s,e,L) = \exp\left(\frac{L-RECL50\%(s,e)}{RECL75\%(s,e)-RECL50\%(s,e)}\log 3\right)$$

Thus, Eq. (4.2) may be extended to take into account recruitment to fishing grounds by

$$\bar{F}(e,y,s,L) = EF(e,y)\,\frac{GSEL(s,e,L)}{1+GSEL(s,e,L)}\cdot\frac{1}{1+RGSEL(s,e,L)}\cdot\frac{REC(s,e,L)}{1+REC(s,e,L)}$$

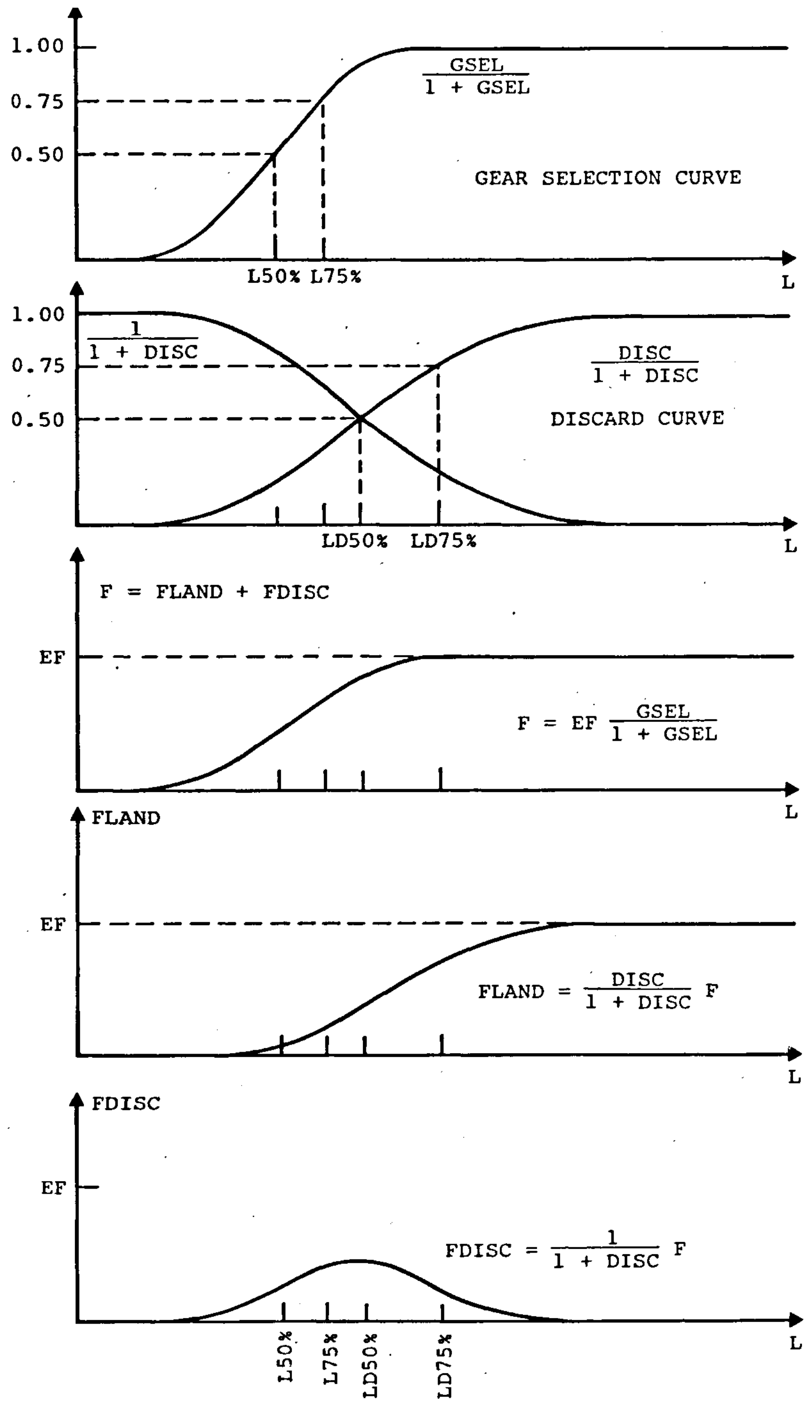
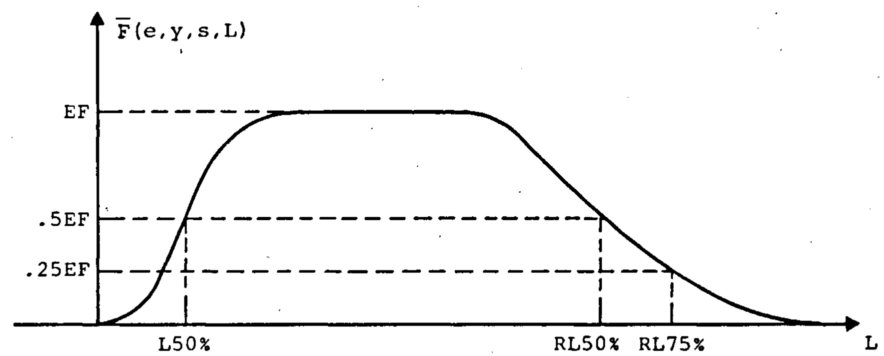


Figure 4. Fishing mortality as a function of length, and fishing mortality partitioned into discard- and landing fishing mortalities.

Figure 5: Right hand side descending selection curve.



4.2 TECHNICAL INTERACTION

"Technical interaction" or "mixed fisheries" means that the effort exerted by a fleet (usually) produces fishing mortality on a number of stocks.

Thus, total fishing mortality on a stock is (usually) the sum of a number of components, coming from various fleets fishing on the stock in question.

The definition of a "fishing fleet" concept is far from obvious. The simplest approach is that adopted by the North Sea round fish W.G.(Anon.,1980) where the total fleet is divided into a consumption fleet and an industrial fleet. The next step into a further classification could be to divide into national fleets and then divide the national fleets into smaller units characterized by vessel- and gear type, fishing grounds and catch compositions.

The problem of defining an appropriate fleet concept is not attempted solved in the present work.

In the following it is assumed that a division of the total international fleet into management units exists.

Bycatch distributions are defined by the matrix

$$BYC(e,s)$$

where e is index of fleet and s is index of fleet.

If s is target species of fleet e, then BYC(e,s)=1.0 by definition.
If s is bycatch species of fleet e, then bycatch fishing mortality is defined

$$FBYC(e,y,s,a) =$$

$$BYC(e,s)EF(e,y)\ \frac{GSEL(s,e,L(s,a))}{1+GSEL(s,e,L(s,a))} \cdot \frac{1}{1+RGSEL(s,e,L(s,a))} \cdot \frac{REC(s,e,L(s,a))}{1+REC(s,e,L(s,a))} \tag{4.3}$$

where GSEL for bycatch species is defined as for the target species:

$$GSEL(s,e,L(s,a)) = \exp\left(\frac{L(s,a)-L50\%(s,e)}{L75\%(s,e)-L50\%(s,e)}\ \log 3\right)$$

By definition FBYC(e,y,s,a) = F(e,y,s,a) if s is target species of fleet e.

Assuming the three right hand terms of Es. (4.3) to be 1.0, i.e. that agegroup a is at maximum exploitation and assuming that age group a of the target species is also fully exploited then

$$FBYC(e,y,s,a) = BYC(e,s) \cdot F(e,y,j,a)$$

where j is index of target species.

Thus, in this case BYC fulfils the equation (see Figure 6)

$$BYC(e,s) = \frac{FBYC(e,y,s,a)}{F(e,y,j,a)} = \frac{\text{fishing mortality on the bycatch species}}{\text{fishing mortality on the target species}} \tag{4.4}$$

Figure 7: Fishing mortalities on one species (s) fished by 3 fleets

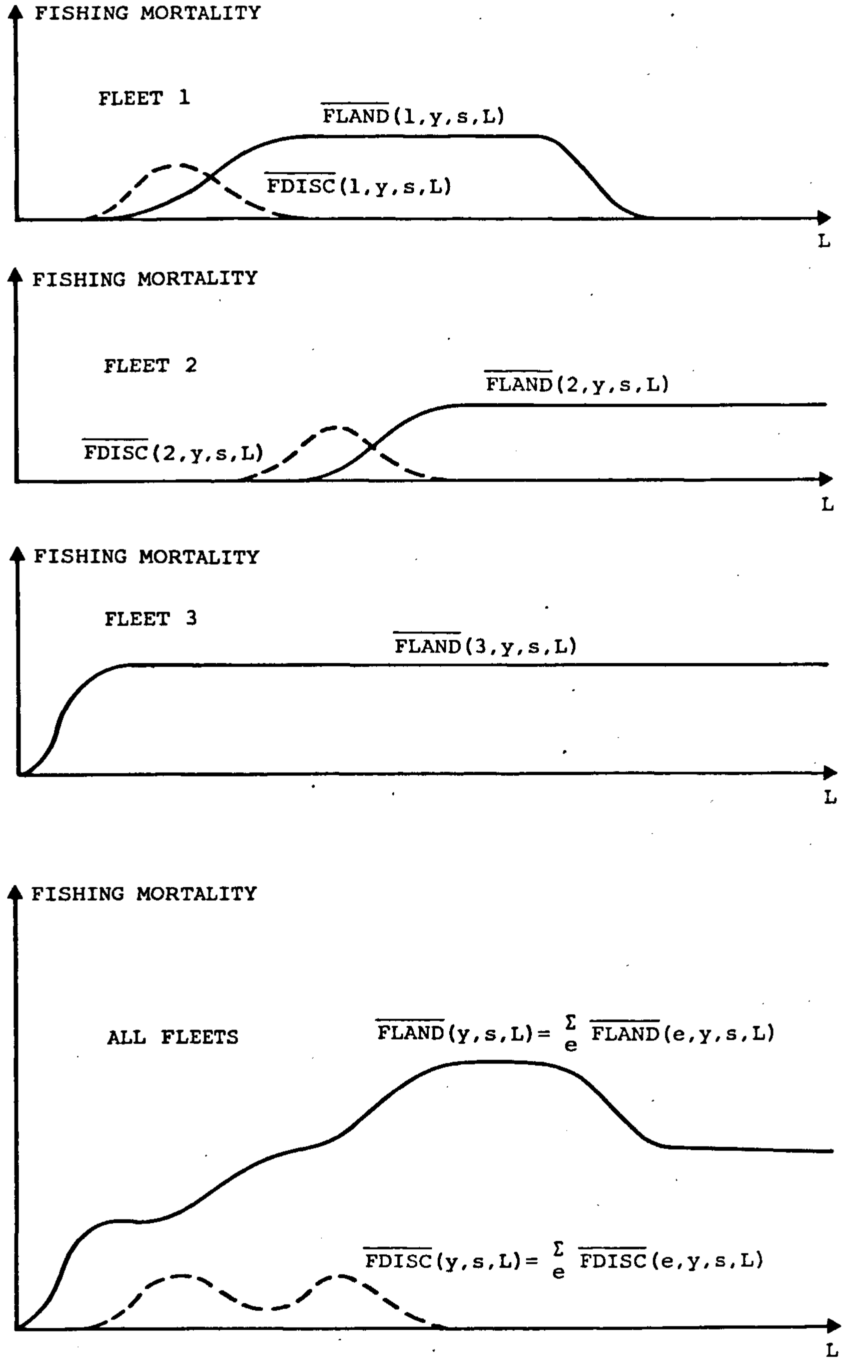
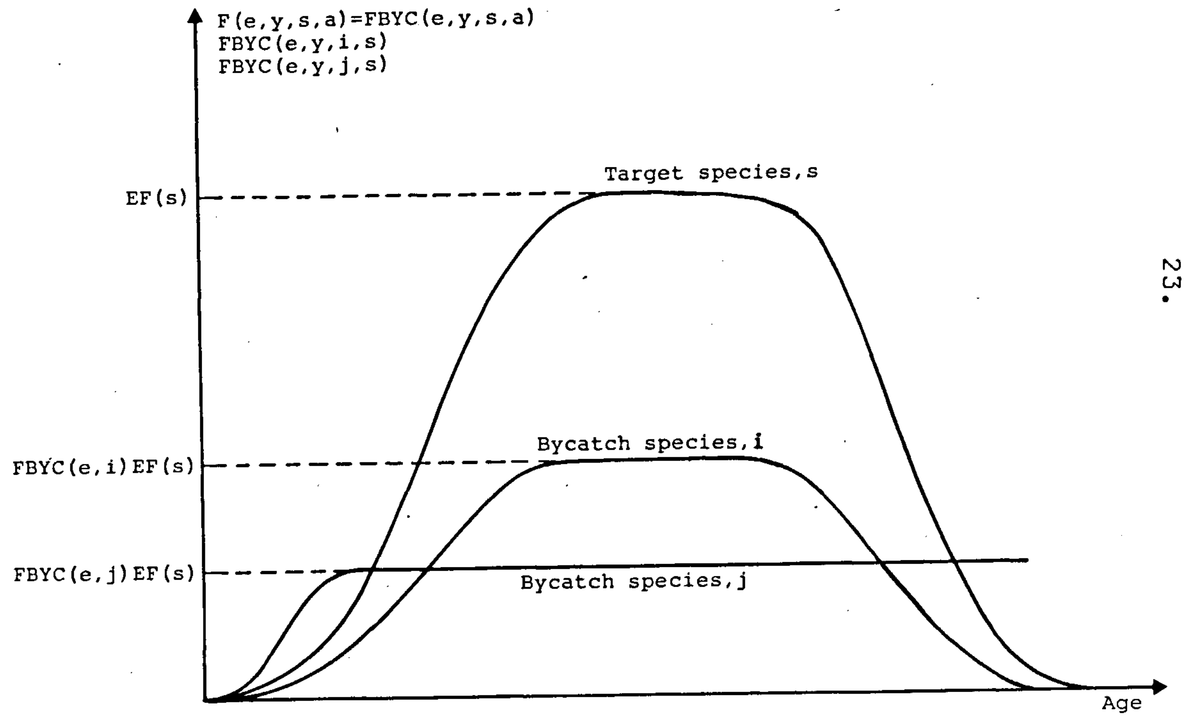


Figure 6: Fishing mortalities on target species (s) and two bycatch species (i and j) BYC(e,s)=1.0, BYC(e,i)=.47 and BYC(e,j)= 0.26



If the two species s and j (bycatch species and target species, resp.) have the same selection curve, then Eq. (4.4) holds for all agegroups.

To sum up: Technical interaction is given by the bycatch matrix:

Species / fleet	1	2	.....	S
1	BYC(1,1)	BYC(1,2)	.....	BYC(1,S)
2	BYC(2,1)	BYC(2,2)	.....	BYC(2,S)
⋮	⋮	⋮		⋮
E	BYC(E,1)	BYC(E,2)	.....	BYC(E,S)

If total number caught by each fleet is known then BYC may be estimated by:

$$BYC(e,s) = \frac{\text{number caught by fleet e}}{\text{Total number caught}}$$

"number caught" only refers to agegroups of maximum exploitation.

Landing and discard fishing mortalities on bycatch species are defined:

$$FBLAND(e,y,s,a) = FBYC(e,y,s,a)\ \frac{DISC(s,e,L(s,a))}{1+DISC(s,e,L(s,a))}$$

$$FBDISC(e,y,s,a) = FBYC(e,y,s,a)\ \frac{1}{1+DISC(s,e,L(s,a))}$$

Total fishing mortalities of species s agegroup a in year y are:

$$F(y,s,a) = \sum_e FBYC(e,y,s,a)$$

$$FLAND(y,s,a) = \sum_e FBLAND(e,y,s,a)$$

$$FDISC(y,s,a) = \sum_e FBLAND(e,y,s,a)$$

(see Figure 7)

Thus, from the bycatch matrix and the gear selection parameters, matrices for the landing and discard mortalities can be derived (see Table 1).

Species / fleet	Species 1				Species S		
	age gr. 1.	age gr. 2.	...	...	age gr. 1.	age gr. 2.	...
1	FBLAND(1,y,1,1)	FBLAND(1,y,1,2)	...	...	FBLAND(1,y,S,1)	FBLAND(1,y,S,2)	...
2	FBLAND(2,y,1,1)	FBLAND(2,y,1,2)	...	...	FBLAND(2,y,S,1)	FBLAND(2,y,S,2)	...
⋮	⋮	⋮			⋮	⋮	
E	FBLAND(E,y,1,1)	FBLAND(E,y,1,2)	...	...	FBLAND(E,y,S,1)	FBLAND(E,y,S,2)	...
TOTAL	FLAND (y,1,1)	FLAND (y,1,2)	...	...	FLAND (y,S,1)	FLAND (y,S,2)	...

Species / fleet	Species 1				Species S		
	age gr. 1.	age gr.2.	...	...	age gr. 1	age gr.2.	...
1	FBDISC(1,y,1,1)	FBDISC(1,y,1,2)	...	...	FBDISC(1,y,S,1)	FBDISC(1,y,S,2)	...
2	FBDISC(2,y,1,1)	FBDISC(2,y,1,2)	...	...	FBDISC(2,y,S,1)	FBDISC(2,y,S,2)	...
⋮	⋮	⋮			⋮	⋮	
E	FBDISC(E,y,1,1)	FBDISC(E,y,1,2)	...	...	FBDISC(E,y,S,1)	FBDISC(E,y,S,2)	...
TOTAL	FDISC (y,1,1)	FDISC(y,1,2)	...	...	FDISC (y,S,1)	FDISC (y,S,2)	...

TABLE 1. Symbolic landing and discard mortality matrices

5. PROGNOSIS

The ordinary single species procedure is to consider the fishing mortalities of future years as decision variables, and to assume recruitment and natural mortalities of future years to be known, and then to continue the VPA calculation scheme into future years. The procedure is straightforward and is based on the wellknown formulas:

$$N(y+1,s,a+1) = N(y,s,a)\exp(-Z(y,s,a))$$

$$C(y,s,a) = F(y,s,a)\ \overline{N}(y,s,a)$$

In the ordinary single species application the calculations are less extensive compared to those of VPA, since we don't need to solve any equation in F. The new thing in species interaction prognosis is the partitioning of Z into the three sources of deaths Z = M1 + M2 + F. M1 is assumed to be known and M2 is calculated as described in section 3.1. (Eq.3.4).

For a detailed description of the prognosis procedure see Appendix D.

If the number of fleets equals the number of species considered and if we put

$$BYC(e,j) = \begin{cases} 1 & \text{if } j \text{ is target species of fleet } e \\ 0 & \text{if } j \text{ is not target species of fleet } e \end{cases}$$

then the prognosis model reduces to the ordinary single fleet prognosis procedure.

If further all SUITs are put equal to zero (cf. section 3.1) we end up with the traditional single species catch prediction procedure usually applied by ICES working groups. Notice that ordinary mesh assessment can be performed by the present method.

5.1 SHORT TERM PROGNOSIS

Let LASTY be the last year for which catch statistics are available. The"short term prognosis" (or the tactical application of the model) refers to the situation in year LASTY +1 where an ICES working group is going to advise on the TAC for year LASTY +2.

Assuming that the ACFM has decided what the strategy for the long term exploitation of the stocks should be, there are virtually no new problems running the legion analysis in the forecast mode.

Recruitments N(LASTY +1,s,YAGE(s)), s=1,2,.... S in the"present year" (the year of the W.G.meeting) are oftenly known from young fish surveys. Recruitments of the year for which the TAC is to be determined N(LASTY+2,s,YAGE(s)) is usually of little importance for the catch quotas. Most likely N(LASTY+2,s,YAGE(s)) will be estimated by the average recruitment for,say, the last 10 years.

The strategic problem: Should TACs be increased, reduced or remain unchanged compared to last year's catch ? is not solved by the tactical approach.

5.2. LONG TERM PROGNOSIS.

A strategy for the long term exploitation of fish stocks, is the necessary basis for a meaningfull TAC determination. A long term strategy can be assessed simply by running the legion analysis in the forecast model for, say, 25 years. Most likely, we will assume fishing patterns to remain constant in all 25 years. In this long term application the stock/recruitment relationship becomes one of the dominant mechanisms of the model.

I expect that we want to run the prognosis for as many years as necessary for the system to run into a steady state situation, under given fishing patterns.

Steady state implies that recruitment is constant as a function of the ecosystem (i.e. as a function of spawning stock biomass and abundance of predators on the juveniles). As the model may be partitioned into a juvenile-model (describing the first year of life, cf. section 3.5) and a model of the adult life, there are two recruitment concepts

NO(y,s,1): recruitment to the juvenile stage (third index, 1, refer to the first time period, cf. Fig. 2.)

N(y,s,YAGE(s)): recruitment to the adult stage model (At Jan. 1.).

N(y,s,YAGE(s)) is the recruitment concept usually applied by ICES WGs. So compared with traditional models, this approach takes into account predation (e.g. cannibalism) in the stock recruitment model.

The number of deaths during the first year of life (from "birthday" to Jan. 1.)

NO(y,s,1)-N(y,s,YAGE(s))

is (partly) determined by predation. This feature should be taken into consideration when choosing a stock/recruitment model. A Ricker type of stock/recruitment curve is dubious in this model because the compensatory effect is already built into the model.

The stock/recruitment model applied in the present version of legion analysis, is essential one in which the recruitment is nearly constant, (that is, NO is nearly independent of spawning stock biomass) unless the spawning stock biomass approaches zero. The model is of the Beverton and Holt type (Beverton and Holt, 1956)

$$NO(y,s,1) = NOMAX(s)\,\frac{EGG(y,s)}{HALFSAT(s)+EGG(y,s)}$$

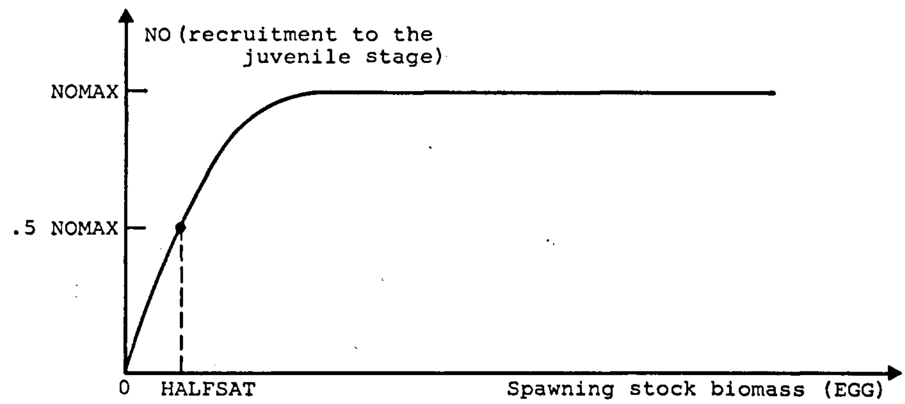
where "EGG" is a function of spawning stock biomass

$$EGG(y,s) = \sum_{a \geq MAGE(s)} N(y,s,a)\,\overline{W}(s,a)\;SPAW(s,a)$$

The coefficients "SPAW" are constant parameters.

SPAW(s,a) may be interpreted as the number of hatching larvae per kg spawning stock (agegroup a) and consequently EEG may be inter-

Figure 8: Stock/recruitment model.



preted as the total number of hatching larvae in year y.

Figure 8 shows a typical stock/recruitment curve.

HALFSAT(s) is the half saturation constant which determines the steepnes of the left hand side of the curve.

The parameter NOMAX(s) is the maximum number of recruits.

To consider N(y,s,YAGE(s)) as a function of only spawning stock biomass has little sense since N(y,s,YAGE(s)) is a function of entire ecosystem.

The stock recruitment problem is not supposed to be solved by the legion analysis. However, it is hoped that a part of it is approached by the inclusion of predation mortality in early life of fish.

A sound approach to the stock recruitment problem may be to consider recruitment as a stochastic process

N(y,s,YAGE(s))= NO(y,s,1) + (stochastic term)

NO is a function of spawning stock only.

The stochastic term accounts for the number of deaths during the juvenile period.

The stochastic term is a function of the entire ecosystem (temperature, currents, abundance of food animals, abundance of predators etc.). It has no sense to press this extremely complicated problem into the frame of a two dimensional coordinate system.

The stochastic term may be devided into terms accounting for various sources of influence from the ecosystem, e.g.

(stochastic term)=

(predation induced deaths) +
(starvation induced deaths) +
(disease induced deaths) +
(residual stochastic term)

In legion analysis the stochastic term is divided into two:

(predation induced deaths)
+ (residual stochastic term)

and an attempt to estimate the expected value of predation induced deaths is made. Is is hoped that this approach will reduce the variance of the stochastic term.

Accepting that recruitment is a stochastic process an advisable approach is that of N.A. Nielsen (1979) where the stochastic term is drawn from a random number generator. Then, by aid of stochastic simulation techniques, the distributions of various variables (stock sizes, catches etc.) are derived. In principle the model developed by N.A. Nielsen can be applied to the present model.

6. A GOAL FUNCTION FOR THE ENTIRE INTERNATIONAL FISHERY.

This section deals with the evaluation of the various predictions made by the prognosis program. YLAST is the last year for which catches are known, and year YLAST+1 is the first year for which prognosis is made. The years

YLAST+1, YLAST+2,... , YFOR

are the future years we consider, and in the following the index 'y'refers to a future year ($\text{YLAST}+1 \leq y \leq \text{YFOR}$).

Because some (rather superficial) economic considerations will be done, biomass must be related to money (cf. Gulland 1979) and to relate money to particular years a rate of interest, r, must be introduced. A capital K(y) in year y is given the value $K(y)(1+r)^{-(y-\text{YLAST})}$ in year YLAST. And the value of productions (measured in capital units)

K(YLAST+1), K(YLAST+2), ... , K(YFOR)

in the years YLAST+1,YLAST+2, ... , YFOR is defined by

$$\sum_{y=\text{YLAST}+1}^{\text{YFOR}} K(y)(1+r)^{-(y-\text{YLAST})} .$$

The yield of fleet e, YIELD(y,e,s,a) from species s agegroup a in year y is

$$\text{YIELD}(y,e,s,a) = \text{FLAND}(y,e,s,a)\ \overline{N}(y,s,a)\ \overline{w}(s,a)$$

Total yield of fleet e during year y is

$$\text{YIELD}(y,e) = \sum_{s}\sum_{a} \text{YIELD}(y,e,s,a)$$

whereas the total yield from species s agegroup a caught by all fleets is $Y(y,s,a) = \sum_{e} \text{YIELD}(y,e,s,a)$.

If we introduce a new concept "value" or "return-value", V(y,e,s,a), of the yield of fleet e, per kilo fish caught of species s agegroup a in year y, we can talk about the value of the catch, i.e. the value of YIELD(y,e,s,a) in year y is

$$V(y,e,s,a)\ YIELD(y,e,s,a)$$

and the return in year LASTY is

$$V(y,e,s,a)\ YIELD(y,e,s,a)(1+r)^{-(y-YLAST)}$$

V could e.g. be the expected price per kilo of landings

$$V = PRICE/KG.$$

Another possibility is to define

$$V = PRICE/KG - \frac{PRICE\ PER\ UNIT\ EFFORT}{C.P.U.E.}$$

so that the goal function becomes the net return.

There are a number of difficulties in this approach, but as I consider these as being outside the scope of fishery biology they should be left to economists and administrators.

If V(y,e,s,a) = 1.0 for all indices and r = o the goal function is simply the sum of biomasses of all landings. For a discussion of this goal function compared to the current one applied by the ACFM see App. H.

These specific choices of V are only given as examples. <u>It is not the task of fishery biologists (ICES experts) to advise on the definition of the Vs.</u> The definition of the Vs is a political decision, and V should be defined before the biologists give their advice on management of fisheries. If politicians ask for advice on the choice of evaluation rules for the various products of fishery (e.g. the values of Vs) we are outside the scope of biology. There is no "true scientific definitions" of which value man should put on the various resources of the sea.

The total value of fleet e's catches during the years YLAST+1, ... , YFOR of species s agegroup a is

$$\sum_y V(y,e,s,a)\ YIELD(y,e,s,a)(1+r)^{-(y-LASTY)}$$

the total value of fleet e's catches of all species and all agegroups is

$$\sum_y \sum_s \sum_a V(y,e,s,a)\ YIELD(y,e,s,a)(1+r)^{-(y-LASTY)}$$

The return value of the total international yield during the years from YLAST+1 to YFOR is

$$RETURN = \sum_e \sum_y \sum_s \sum_a V(y,e,s,a)\ YIELD(y,e,s,a)(1+r)^{-(y-LAST)}$$

RETURN depends on the choice of efforts, gear selection regulations and bycatch regulations. That is

$$RETURN = RETURN\ (\underline{F})$$

where $\underline{F}$ stands for the set of fishing mortalities

FLAND(y,e,s,a), FDISC(y,e,s,a) e= 1,2,...., E
y= YLAST+1,YLAST+2,...,YFOR
s= 1,2,...,S
a= YAGE(s),YAGE(s)+1,...OAGE(s)

and each pair FLAND,FDISC depends on EF(y,e),L50%(s,e) and L75%(s,e) (for both landings and discards) and BYC(e,j).

Thus RETURN is a function of:

EF: effort (EF is assumed to be proportional to effort)

L50%, L75%:left hand side gear selection (e.g. mesh size)
(RL50%, RL75%):right hand side gear selection
BYC: bycatch regulations
LD50%, LD75%: Discards

BYC is not a pure decision variable, i.e. BYC can only partly be controlled by man (c.f. section 4).

We are now able to give the first simple definition of the central problem in fishery management:

$$\boxed{\text{Determine } \underline{F}\text{, so that RETURN } (\underline{F}) \text{ is maximized}} \qquad (6.1)$$

This somewhat primitive formulation may have certain shortcomings The solution of (6.1) may turn out to be one in which all stocks are depleted at the end of year YFOR.

To avoid depletion of stocks, it may be natural to introduce certain constraints on (6.1)which could prevent the stocks from depletion:

SSB(y,s) > MINSSB(s) for all y.

where MINSSB(s) stands for "minimum allowable spawning stock biomass" of species s. An optimum (theoretical) solution of (6.1)may also imply that effort is raised to a level above what is physically possible (simply because of a limited number of vessels). Thus, another natural constraint to be put on (6.1) is

EF(y,e) < MAXEF(y,e)

where MAXEF stands for "maximum number of effort units available to fleet e". (EF and MAXEF are assumed to be proportional to effort). Due to social and economic regards we may wish to enforce the constraint on the system that certain fleets should not be forced to stop essential parts of their activities. This constraint could be formulated

MINEF(y,e) < EF(y,e) < MAXEF(y,e).

This constraint about the distribution of effort units could have been formulated in a way which would allow vessels to change from one fishery to another, but for the moment this aspect is ignored.

The problem of how effort units should be defined is not attempted solved in the present work.

Including the constraints we then arrive at the more detailed definition of the central problem:

Determine $\underline{F}$ so that RETURN ($\underline{F}$) is maximized under the constraints:
SSB(y,s) > MINSSB(s)   for all y and s   (I)
MAXEF(y,e) > EF(y,e) > MINEF(y,e) for all e   (II)

The two constraints (I) and (II) may result in inconsistencies. If e.g. $\underline{F}$ is found to be optimum at a high level, (I) may be impossible to fulfill, so either (I) or (II) should be given a higher priority than the other.

The program developed so far is able to calculate RETURN($\underline{F}$) and the optimum value can only be approached by the trial and error method. No real optimization algorithm has been developed.

It may also be questioned whether the concept of "optimum solution" is defineable in the case of fishery management for a longer period of future years. Rather than searching for one optimum solution I feel that a range of solutions should be considered for a range of goal functions.

For example, it could be decided that three goal functions should be considered:

	V(y,e,s,a)	r	GOAL FUNCTION $\sum_{e}\sum_{y}\sum_{s}\sum_{a} V \cdot YIELD(1+r)^{-(y-LASTY)}$
1	1.0	0	TOTAL BIOMASS LANDED
2	PRICE PER KG LANDED	0	TOTAL RETURN FROM SALE OF ALL LANDINGS
3	PRICE PER KG LANDED MINUS EXPENCES PER KG LANDED	0	TOTAL NET RETURN OF ALL LANDINGS

If it is decided that we are not so concerned about what happens in the far future as what happens the next few years, r should be given a positive value. For each of these three alternative goal functions, a number of alternative fishing strategies should be considered.

For example:

strategy 1: Effort of all fleets remains constant.

strategy 2: Effort of all fleets reduced by 10 % in all future years.

strategy 3: Effort of all fleets fishing for gadoid fish reduced by 10 percent. Effort of other fleets remains unchanged.

strategy 4: Effort of all fleets fishing for gadoid fish reduced by 10 percent. Effort on fleets fishing for plaice increased by 10 %.Other fleets unchanged.

.
.
.
.
etc.

The above set of goal functions and alternative fishing strategies is given only as a (hypothetical)illustration of the ideas.

7. DISCUSSION.

As demonstrated by Macer, Jones and Bannister, 1979 the current catch predictions based on the traditional methods should be treated with a certain reservation.

Some of these difficulties are hoped to be overcome by the model suggested in this paper.

However, there are problems which cannot be solved by improving the theoretical basis of assessment. The limited succes of the traditional assessment methods is caused by two main reasons:

1) The single species/single fleet model is a too rough approximation of reality.

2) The data base used for assessment has been incomplete, biased and (or) badly understood.

Due to the shortcomings of current data bases it is actually not possible to give a proper evaluation of the single species/single fleet model.

For the short term prognosis the single species approach may be a reasonable tool for setting TACs, if the necessary data base were available.

The crucial parameters for the short term prognosis are the fishing mortalities for the last year for which catches are reported. (the final Fs).

The final Fs can be estimated from effort data, if the catchability coefficients are known. However, usually little is known about catchability coefficients.

This paper does not suggest a solution to the problem of estimation of catchability coefficients. That is, the parameters EF(e,y) (cf. section 4.1) are not expressed as a function of, say, number of trawl hours, engine power, size of vessels etc..

As for the traditional methods, the applicability of the present model is rather limited before the catchability coefficient problem has found a reasonable solution. It is hoped that the partitioning of the fleets in management units, as suggested in this work, will make it easier to determine the catchability coefficients.

Data on discards and industrial bycatches are usually incomplete and always determined with a larger uncertainty than the landings for human consumptions. As discards and industrial bycatches consist in younger fish, these may account for large proportions of the number caught even if the weight of these components are relatively small.

It can be (and it ought to be) discussed whether ICES WGs are in a position to give advice on TAC with the current level of data collection. If actually the prognosis for the stock size has an uncertainty of, say, 100 % (coeff. of variance) one single TAC-value is meaningless. For some threatened stocks it is obvious to everybody that TACs should be enforced, but for the remaining non threatened stocks the TAC must be considered as a more or less random number.

As a consequence of the above, I tend to consider the present contribution as an introduction to a discussion of what data base is needed to improve the assessment made by ICES WGs.

It is an important step forward to start the international stomac sampling sheme in 1981, but it is not enough.

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APPENDIX A.

CALCULATION OF FISHING MORTALITY WHEN NUMBER OF PREDATION INDUCED DEATHS IS KNOWN.

The two ordinary VPA equations are

$$N(y+1,s,a+1) = N(y,s,a)\exp(-Z(y,s,a))$$

$$C(y,s,a) = N(y,s,a)F(y,s,a)(1-\exp(-Z(y,s,a)))/Z(y,s,a) \quad (A1)$$

These two wellknown equations are also solved in legion analysis, but the equations are rewritten as follows:

$$Z(y,s,a)=F(y,s,a)+M1(y,s,a)+M2(y,s,a)=F(y,s,a)PHI(y,s,a)+M1(y,s,a)$$

where $PHI(y,s,a) = 1+D(y,s,a)/C(y,s,a)$, which follows

from $D(y,s,a) = M2(y,s,a)N(y,s,a)(1-\exp(-Z(y,s,a)))/Z(y,s,a)$

so that $M1(y,s,a)/F(y,s,a) = D(y,s,a)/C(y,s,a)$.

Inserting the new expression for Z into Eq (A1) and rearranging the terms gives

$$\frac{F(y,s,a)(\exp(F(y,s,a)PHI(y,s,a)+M1(y,s,a))-1)}{F(y,s,a)PHI(y,s,a)+M1(y,s,a)} - \frac{C(y,s,a)}{N(y+1,s,a+1)}=0 \quad (A2)$$

When D is known the unknown variable in Eq (A2) is F(y,s,a).

To facilitate notation we put $X = F(y,s,a)$ and

$G = C(y,s,a)/N(y+1,s,a+1)$ and rewrite Eq (A2) in the form

$$f(X) = X(\exp(PHI \cdot X+M1)-1)/(PHI\cdot X+M1)-G = 0 \quad (A3)$$

where f stands for "function". Thus we want to solve the equation

$$f(X) = 0$$

This can be done e.g. by aid of the Newton iteration procedure, which generates a serie $X_1, X_2, \dots, X_n$, .. This infinite serie converges (usually) against the solution to $f(X)=0$. X_n is found from

$$X_n := X_{n-1} + f(X_{n-1})/f'(X_{n-1}) \quad (A4)$$

From a differentiation of Eq. (A3) it follows that

$$\frac{f}{f'} = \frac{F(\exp(Z)-1)-ZG}{(M1/Z+F\cdot PHI)\exp(Z)-M/Z}$$

To start the serie of solutions of pairs of equations we still need to start with a guess on the final F's, and for the last year we need to know F for all age groups, as in ordinary single species VPA.
In principle the procedure described above is exactly the same as that used in ordinary single species VPA.

APPENDIX B.

LEGION ANALYSIS.

The two basic equations of ordinary single species VPA are:

$$N(y+1,s,a+1) = N(y,s,a)\exp(-Z(y,s,a)) \qquad (B1)$$

$$C(y,s,a) = F(y,s,a)N(y,s,a)(1-\exp(-Z(y,s,a)))/Z(y,s,a) \qquad (B2)$$

where

$N(y,s,a)$ is the number of a year old fish in the beginning of year y from species s (i.e. the number of survivors in the sea of yearclass y-a in the beginning of year y). Because (B1) refer to a singlespecies model the index s could have been omitted.

$C(y,s,a)$ is the number caught during year y.

$F(y,s,a)$ is the fishing mortality. F is assumed to remain constant during year y.

$Z(y,s,a)$ is the total mortality in year y. Z is assumed to remain constant during year y. $Z(y,s,a) = F(y,s,a) + M(y,s,a)$ where

$M(y,s,a)$ is the natural mortality in year y. M is assumed to remain constant during year y.

$\overline{N}(y,s,a)=N(y,s,a)(1 - \exp(-Z(y,s,a)))/Z(y,s,a)$ is the average number of survivors in year y. (B3)

Inserting $\overline{N}(y,s,a)$ into (B2) we get

$$C(y,s,a) = F(y,s,a)\overline{N}(y,s,a) \qquad (B4)$$

Ordinary VPA is usually carried out as a serie of separate calculations for a number of yearclasses. The procedure is illustrated in Table B1, where the equations to be solved for yearclass 1970 (from some hypothetical species) are shown for the first four age groups. The equations of Table B1 are (B1) and (B4). The unknown variables are the F´s and the N´s. C is known and M is assumed also to be known. Actually, no one knows anything exactly about M, for mathematical reasons we have to make assumptions about M, since we were otherwise unable to determine a unique solution to the equations.

There are two equations for each year, and there are two unknown variables (F and M) for each year plus one extra unknown, namely N for the oldest agegroup. That is, we have 2n equations but 2n+1 unknowns, where n is the number of agegroups considered. That means that we are still unable to find a unique solution. The problem is usually "solved" by making a guess on one of the 2n+1 unknowns. Usually a guess is made on F for the oldest age group.

If we consider the M´s as unknown variables (which they actually are) the status of ordinary VPA can be summarized:

age	1970	1971	1972	1973
1	C(70,s,1)=F(70,s,1)$\overline{N}$(70,s,1) N(70,s,1)= N(71,s,2)exp(Z(70,s,1))	71,s,1	72,s,1	73,s,1
2	70,s,2	C(71,s,2)=F(71,s,2)$\overline{N}$(71,s,2) N(71,s,2)= N(72,s,3)exp(Z(71,s,2))	72,s,2	73,s,2
3	70,s,3	71,s,3	C(72,s,3)=F(72,s,3)$\overline{N}$(72,s,3) N(72,s,3)= N(73,s,4)exp(Z(72,s,3))	73,s,3
4	70,s,4	71,s,4	72,s,4	C(73,s,4)=F(73,s,4)$\overline{N}$(73,s,4) N(73,s,4)= N(74,s,5)exp(Z(73,s,4))

Table B1. The calculational procedure of traditional single species VPA. The arrows indicate the chronological order of the calculations.

1	D(70,s,1)=M2(70,s,1)$\overline{N}$(70,s,1) C(70,s,1)=F(70,s,1)$\overline{N}$(70,s,1) N(70,s,1)= N(71,s,2)exp(Z(70,s,1))	71,s,1	72,s,1	73,s,1
2	70,s,2	D(71,s,2)=M2(71,s,2)$\overline{N}$(71,s,2) C(71,s,2)=F(71,s,2)$\overline{N}$(71,s,2) N(71,s,2)= N(72,s,3)exp(Z(71,s,2))	72,s,2	73,s,2
3	70,s,3	71,s,3	D(72,s,3)=M2(72,s,3)$\overline{N}$(72,s,3) C(72,s,3)=F(72,s,3)$\overline{N}$(72,s,3) N(72,s,3)= N(73,s,4)exp(Z(72,s,3))	73,s,3
4	70,s,4	71,s,4	72,s,4	D(73,s,4)=M2(73,s,4)$\overline{N}$(73,s,4) C(73,s,4)=F(73,s,4)$\overline{N}$(73,s,4) N(73,s,4)= N(74,s,5)exp(Z(73,s,4))

Table B2. Legion analysis calculational procedure for one stock. Notice that this calculation is dependent on the corresponding calculations for all other considered stocks. Legion analysis is performed on a yearly basis as indicated by the arrows.

3n+1 unknown variables (F,M and N). n is the number of age groups.

2n equations.

n+1 variables are determined by guesswork.

2n variables are determined by solving the equations (N and F).

Actually, the implication is that you can get any result you want out of a VPA. Thus, it can be discussed whether VPA is an art or a scientific method.

The usual calculational procedure in ordinary VPA is indicated with arrows in Table B 1. You start by solving the equations for the four year old in 1973 (F(1973,s.4) is guessed). N(1973,s.4) is then used to determine F and N for the three year old in 1972, ... etc.

Legion analysis is to be considered an extension to the ordinary VPA. There is still a large amount of quesswork in legion analysis. The number of unknown variables compared to the number of equations is only slightly reduced. The advantage of legion analysis is that only a part of M has to be guessed.

The new thing in legion analysis is the introduction of an extra equation and an extra unknown variable for each year considered.

The three equations are:

$$N(y+1,s,a+1) = N(y,s,a)\exp(-Z(y,s,a)) \quad \text{(B1)}$$
$$C(y,s,a) = F(y,s,a)\overline{N}(y,s,a) \quad \text{(B4)}$$
$$D(y,s,a) = M2(y,s,a)\overline{N}(y,s,a) \quad \text{(B5)}$$

where D(y,s,a) is the number of fish devoured by predators during year y. The new variable is M2, which stands for "predation induced natural mortality". That is, M is partitioned into two quantities

$$M = M1 + M2$$

where M1 stands for "other" natural mortality (i.e. disease, starvation, spawning stress, etc.).

M is still found by pure guesswork, but M2 is estimated except for the oldest age group. The term "legion" is applied because M2 is estimated from a multispecies assessment on the predators of the species considered. (A "legion" consists of a number of "cohorts"). In single species VPA, one only needs to consider a single yearclass at a time. What happens to the rest of the stock or the rest of the ecosystem has no influence on the results for that particular yearclass. With other words, what happens in the blank boxes of Table B 1 is indifferent to the result for yearclass 1970. In legion analysis it is essential to every yearclass what happens in all the yearclasses of its predators. This feature implies that legion analysis should be carried out on a yearly basis rather than on a yearclass basis. The year to year nature of legion analysis is indicated by arrows in Table B 2. The backwards step from one year to the preceding year in the calculational procedure must be carried out simultaneously for all age groups of all species. (See Table B 3).

If we for a moment assume M2 to be known then the calculational procedure for determination of F and N is the same as that applied in ordinary VPA. A detailed description of the procedure is given in Appendix A.

The estimation of F, M2 and N in legion analysis is an iterative proces:

1. Make an initial quess on M2 (e.g. M2=0)

2. Calculate F and N, (as in ordinary single species VPA)

3. Calculate a new value of M2, based on the N's calculated in step 2.

4. If the last calculated value of Z (=M1+M2+F) deviates more than a certain amount from the value of Z calculated in the preceding iteration, then go to 2.

5. FINIS.

The iterative procedure above refers to a single backwards step between two years. In Figure B 1 this procedure is given a symbolic illustration. M2 is calculated from formula (B 5):

$$M2(y,s,a) = D(y,s,a)/\overline{N}(y,s,a) \quad \text{or}$$
$$M2(y,s,a) = D(y,s,a)\,\overline{w}(s,a)/(\overline{N}(y,s,a)\,\overline{w}(s,a))$$

The biomass of devoured fish $D(y,s,a)\overline{w}(s,a)$ is calculated as the sum of the quantity eaten by each predator

$$D(y,s,a)\,\overline{w}(s,a) = \sum_j \sum_b \begin{pmatrix}\text{predator } (j,b)'\text{s consump-}\\ \text{tion of prey } (s,a).\end{pmatrix} \qquad (B6)$$

where (j,b) means "species j's age group b". Terms of the sum may be zero, and usually more than fifty percent of the terms are zero. Due to notational convenience all species are considered as prey for all other species. For instance the term for sandeel's predation on 8 year old cod is zero.

Predator (j,b)'s total consumption per fish per year is assumed to remain constant from year to year. Thus, density dependent changes in growth rate are assumed to be negligible. The predators always get what they need in one way or another. But the diet composition changes from year to year according to the composition of available food.

The total consumption per year per fish is designated FOOD(j,b).

(B 6) may then be rewritten:

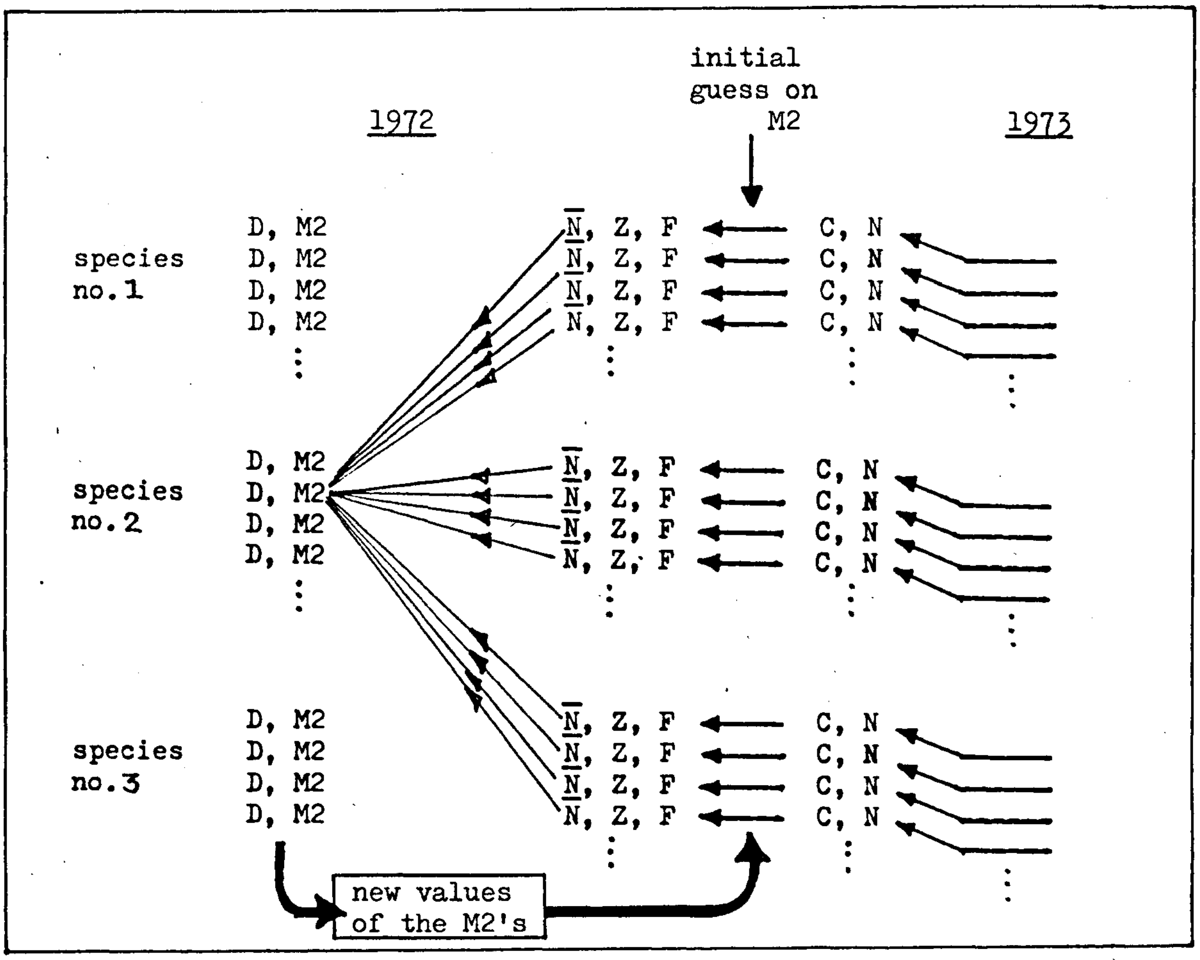

Figure B1. The iterative procedure of legion analysis within a particular year. Actually there should have been an arrow from every N to each M2, but due to good layout this has only been done for a single M2. (An arrow symbolizes a calculational operation).

1970	1971	1972	1973	
70,1,1	71,1,1	72,1,1	73,1,1	
70,1,2	71,1,2	72,1,2	73,1,2	species
70,1,3	71,1,3	72,1,3	73,1,3	no. 1.
70,1,4	71,1,4	72,1,4	73,1,4	
....				
70,2,1	71,2,1	72,2,1	73,2,1	
70,2,2	71,2,2	72,2,2	73,2,2	species
70,2,3	71,2,3	72,2,3	73,2,3	no. 2.
70,2,4	71,2,4	72,2,4	73,2,4	
....				
70,3,1	71,3,1	72,3,1	73,3,1	
70,3,2	71,3,2	72,3,2	73,3,2	species
70,3,3	71,3,3	72,3,3	73,3,3	no. 3.
70,3,4	71,3,4	72,3,4	73,3,4	
....				

Table B3. Legion analysis calculational procedure. Each set of indices (y,s,a) symbolizes a set of three equations as in Table B2.

$$D(y,s,a)\overline{w}(s,a) = \sum_j \sum_b \overline{N}(y,j,b)FOOD(j,b) \cdot \begin{pmatrix}\text{the fraction of pre-}\\ \text{dator (j,b)'s food}\\ \text{obtained from prey(s,a)}\end{pmatrix} \qquad (B\ 7)$$

The last factor in the terms of the sum is calculated from:

$$\begin{pmatrix}\text{the fraction of predator (j,b)'s}\\ \text{food obtained from prey (s,a)}\end{pmatrix} = \frac{\text{(biomass of prey (s,a) available to predator (j,b))}}{\text{(total biomass of food available to predator (j,b))}} \qquad (B\ 8)$$

The concept "available food" is essential to the determination of predation mortality. It is perhaps also the most complex part of the present model. In order to establish a realistic food web every type of biomass must be given a weight indicating its value as food for every predator. For instance the biomass of 8 year old cod is not food available to the one year old cod, and consequently it should be given the weight zero, when available food for the one year old cod is calculated. As e.g. two year old sandeels are excellent food for cod, the biomass of two year old sandeels should be given a positive weight when available food for (e.g.) the five year old cod is calculated. The factors by which the various prey biomasses are assigned an index of "suitability" is called "SUIT". Indices (j,b) designates the predator and (s,a) the prey, i.e. SUIT is the suitability of prey species s age group a as prey for predator j agegroup b. SUIT (s,a,j,b) is a positive number between 0 and 1.0, and

$$\sum_s \sum_a SUIT(s,a,j,b) = 1.0$$

<u>Available food</u> means the biomass of the food multiplied by the corresponding SUIT values. SUIT may be estimated from stomach content investigations as described in Appendix C.

Applying the SUITs formula (B8) becomes:

STOCK(s,a,j,b) =

$$\begin{pmatrix}\text{the fraction of predator}\\ \text{(j,b)'s food obtained}\\ \text{from (s,a)}\end{pmatrix} = \frac{SUIT(s,a,j,b)\overline{N}(y,s,a)\overline{w}(s,a)}{\sum_i \sum_d SUIT(i,d,j,b)\overline{N}(y,i,d)\overline{w}(i,d)} \qquad (B9)$$

Notice that (i,d) in the denominator is index of prey. Perhaps formula (B9) is the best definition of SUIT, i.e. we could define the food suitability matrix SUIT as the set of numbers which fulfils (B9).

Inserting (B9) into (B7) gives

$$D(y,s,a)\overline{w}(s,a) = \sum_j \sum_b \overline{N}(y,j,b)FOOD(j,b) \frac{SUIT(s,a,j,b)\overline{N}(y,s,a)\overline{w}(s,a)}{\sum_i \sum_d SUIT(i,d,j,b)N(y,i,d)w(i,d)} \qquad (B10)$$

And finally we get from M2 = $(D\overline{w})/(\overline{Nw})$ that the predation induced natural mortality coefficient is

$$M2(y,s,a)= \sum_j \sum_b \overline{N}(y,j,b)FOOD(j,b) \frac{SUIT(s,a,j,b)}{\sum_i \sum_d SUIT(i,d,j,b)\overline{N}(y,i,d)\overline{w}(i,d)} \qquad (B11)$$

If the animals of the compartment "other food" act as predators on any of the considered fish species, this source of natural mortality must be included in the residual mortality M1.

We are now able to give a detailed description of the legion analysis calculational procedure.

As mentioned above the calculations are carried out by an iterative procedure. As the criterion for stopping the iterations can be used that

$$\sum_s \sum_a (Z(y,s,a) - ZOLD(y,s,a))^2 < EPSILON$$

where ZOLD stands for total mortality calculated in the preceding iteration and Z stands for the value of total mortality obtained in the current iteration.

Total biomass of the ecosystem in the beginning of year y is

$$TOTB(y) = \sum_i^{S+1} \sum_d N(y,i,d)\ \overline{w}(i,d)$$

where S is the number of fish species considered. S+1 is index of the compartment "other food", which is treated as a stock with one agegroup. Individual weight of other food is $\overline{w}(s+1,1)=1.0$. TOTB(y) is assumed to remain constant. To obtain a constant total biomass, the biomass of other food

$$N(y,S+1,1)$$

is ajusted so that TOTB(y) remains constant. That is, after calculation of the N's for the considered fish species, the biomass of other food is obtained from

$$N(y,S+1,1) = TOTB - \sum_{s=1}^{S} \sum_a N(y,s,a)\ \overline{w}(s,a)$$

where TOTB is the constant total biomass. This means that when there is a large biomass of considered fish then the biomass of other food is low and the opposite when few fish are considered.

Below is a concise description of the algorithm in a pseudo computer language, which the author hopes is immediately understandable to readers with a minimum of experience in computer programming.

ALGORITHM FOR LEGION ANALYSIS.

A : y: = LASTY;

B : Make an initial guess on F, Z and N (e.g. by ordinary single species VPA performed on each species);

C : Make an initial guess on D (e.g. D (y,s,a) = 0 for all s,a);

D : Calculate biomass of other food

$$N(y,S+1,1) := TOTB - \sum_{s=1}^{S} \sum_{a} N(y,s,a)\ \overline{w}(s,a) \cdot$$

E : ZOLD: = Z;

F : for every species and agegroup calculate F as described in App. A.I.e. Let PHI(y,s,a) = 1-D(y,s,a)/C(y,s,a) and solve the equation:

$$\frac{F(y,s,a)(\exp(F(y,s,a)PHI(y,s,a) + M1-1)}{F(y,s,a)PHI(y,s,a)+ M1} - \frac{C(y,s,a)}{N(y+1,s,a+1)} = 0$$

with respect to F (e.g. by Newton iteration)

$$Z(y,s,a) := F(y,s,a) \cdot PHI(y,s,a) + M1;$$
$$N(y,s,a) := N(y+1,s,a+1)\ \exp(Z(y,s,a));$$

G : For every species and agegroup calculate the average number

$$\overline{N}(y,s,a) := N(y,s,a)(1 - \exp(-Z(y,s,a)))/Z(y,s,a);$$

H : For every species and age group calculate number of predation induced deaths:

$$D(y,s,a) := \sum_{j} \sum_{b} \overline{N}(y,j,b) FOOD(j,b) \frac{SUIT(s,a,j,b)\ \overline{N}(y,s,a)\overline{w}(s,a)}{\sum_{i} \sum_{a} SUIT(i,d,j,b)\ \overline{N}(y,i,d)\overline{w}(i,d)}$$

I : If $\sum_{s} \sum_{a} (Z(y,s,a) - ZOLD(y,s,a))^2 > EPSILON$ then goto D;

J : Calculate M2 : $M2(y,s,a) := D(y,s,a)/\overline{N}(y,s,a)$;

K : y:=y-1; if y $>$ FIRSTY then go to B ;

FINIS:

APPENDIX C.

ESTIMATION OF FOOD SUITABILITY MATRIX FROM STOMACH CONTENT DATA.

To illustrate the calculational procedure involved in the estimation of PREF from stomach content data a small hypothetical example is constructed. The example deals with three species of 3, 2 and 3 age groups as shown in Table C1. The column $\overline{N}$ is assumed to be known from a legion analysis Table C2 shows the results obtained from a stomach content survey. Thus, Tables C1-2 are the input data.

species	age	$\overline{N}$	$\overline{w}$	$\overline{Nw}$
1	1	200	5	1000
	2	100	50	5000
	3	50	80	4000
2	1	50000	1	50000
	2	20000	5	100000
3	1	1000	5	5000
	2	500	20	10000
	3	100	30	3000
		Total fish biom.		178000
		Other food biom.		822000
		Total biom. of the ecosystem		1000000

Table C1. Output from VPA ($\overline{N}$) necessary for the estimation of SUIT

PREDATOR(j,b)

	j	1			2		3		
s	a / b	1	2	3	1	2	1	2	3
1	1	0	0	.o5	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
2	1	.50	.50	.30	0	.20	.20	.20	.20
	2	0	.30	.40	0	0	0	.10	.30
3	1	0	.10	.15	0	0	0	0	.10
	2	0	0	.05	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
other		.50	.10	.05	1.00	.80	.80	.70	.40

PREY (s,a)

Table C2. Relative average stomach contents , STOC(s,a,j,b).

Table C3 is calculated from Tables C1-2, and SUIT is immediately obtained from Table C3.

The results are given in Table C4.

PREDATOR(j,b)

j			1			2		3			
s	a	b	1	2	3	1	2	1	2	3	$\overline{N}(y,s,a)\overline{w}(s,a)$
1	1		0	0	.05	0	0	0	0	0	1000
	2		0	0	0	0	0	0	0	0	5000
	3		0	0	0	0	0	0	0	0	4000
2	1		.01	.01	.006	0	.004	.004	.004	.004	50000
	2		0	.003	.004	0	0	0	.001	.003	100000
3	1		0	.02	.03	0	0	0	.02	.06	5000
	2		0	0	.005	0	0	0	0	0	10000
	3		0	0	0	0	0	0	0	0	3000
$other X 10^{+5}$			61	12	6	122	97	97	85	49	822000
$total X 10^{+4}$			106	331	951	12	50	50	259	675	1000000

PREY (s,a)

Table C3. $\dfrac{STOC(s,a,j,b)}{\overline{N}(y,s,a)\ \overline{w}(s,a)}1000$

PREDATOR(j,b)

j			1			2		3		
s	a	b	1	2	3	1	2	1	2	3
1	1		0	0	.53	0	0	0	0	0
	2		0	0	0	0	0	0	0	0
	3		0	0	0	0	0	0	00	0
2	1		.94	.30	.06	0	.80	.80	.15	.06
	2		0	.09	.04	0	0	0	.04	.04
1	1		0	.60	.32	0	0	0	.77	.89
	2		0	0	.05	0	0	0	0	0
	3		0	0	0	0	0	0	0	0
other			.06	.01	0	1.00	.20	.20	.04	.01
Total			1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

PREY (s,a)

Table C4. Food suitability matrix. SUIT(s,a,j,b).

APPENDIX D.

Prognosis calculational procedure.

In principle the prognosis procedure is the same as that applied in legion analysis, except that catches, C is input and fishing mortalities, F is output in legion analysis. In the prognosis procedure F is input and C output. Also the stock/recruitment parameters should be given as input to the program.

The calculational procedure written in a pseudo computer language is:

```
A: Calculate fishing mortalities on each species exerted by each fleet (F(e,y,s,a),
   FLAND(e,y,s,a) and FDISC(e,y,s,a) for the years LASTY,LASTY+1,...,FORY.
B: Calculate fishing mortalities on each species exerted by all fleets (F(y,s,a)).
C: Calculate the number of fish at the beginning of year LASTY+1.
   y:=LASTY.
D: y:=y+1
   Calculate year class strength.
   Make an initial guess on M2(y,s,a).
E: Calculate total biomass of fish and biomass of other food.
F: Calculate average numbers (N̄).      ZOLD:=Z
G: Calculate available food and M2(y,s,a).
   Z:=M1 + M2 + F
H: If ∑ ∑(Z(y,s,a)-ZOLD(y,s,a))^2 > EBSILON   then go to E
      s a
I: Calculate D(y,s,a), C(y,s,a) and N(y+1,s,a+1)
J: If y < FORY then go to D
K: Calculate numbers landed and discarded by each fleet for the years LASTY+1,
   LASTY+2,...,FORY.
   Calculate value of goalfunction for each fleet.
FINIS:
```

APPENDIX E.

AN EXAMPLE.

The computer program has been tested on a data set representing all North Sea fish stocks for which catch at age data are available from ICES WG. reports. The amount of paper output produced by the program from the North Sea data file correspond to about 30 percent of all ICES W.G. reports on North Sea fish stocks. This is the reason why I chose to present a hypothetical example. I believe that it is easier to see the general principles in a smaller example.

The computer run is described by some of the print tables. Some tables contain input data, e.g. catch at age data, and some tables contain results, e.g. fishing mortalities from legion analysis. For each print table is specified which figures are input data and which are results. Input data are labeled "INPUT" and results are labeled "OUTPUT".

The present hypothetical example deals with three stocks, which are named cod, herring and plaice. The years considered, the age groups considered and the total biomass of the ecosystem are given in Table E 1.

As the youngest age group considered (cf. Table E 1) is agegroup 1 for all three species, the dynamics of the O-group (cf. section 3.4) is not covered by the example.

Only cod is considered as a predator.

In principle all species should have been considered as predators (cf. Appendix B). However, the computational effort necessary for the calculation of M2 is reduced considerably when some lesser important predators are ignored. The definition of predators is optional, and it is thus possible to consider all species as predators.

Table E 2 presents $\overline{w}(s,a)$, FOOD(s,a) and M1(a). M1 is assumed to remain constant from year to year. Table E 3 shows the values of SUIT(s,a,j,b) for cod.

Table E 4 gives the guesses on F and M2 for oldest agegroup and last year.

Table E 5 shows the number of iterations performed for each of the year considered and the total biomass of the fish species considered. The number of iterations depends on EPSILON (cf. App. B). In the present application EPSILON = .001. The last line of Table E 5 shows the computation time used by the RC8000 computer at the Danish Institute for Fisheries and Marine Research, to carry out the iterations.

Table E 6 shows the number of prey eaten by cod during year 1973. Similar tables can be printed for every year if the user want it. The figures of table E 6.1 are

$$\overline{N}(1973,1,b)\ FOOD(1,b)\ \frac{\overline{N}(1973,s,a)}{\sum_i \sum_d \overline{N}(1973,i,d) SUIT(i,d,1,b)\ \overline{w}(i,d)}$$

where b is the ageof cod, s is index of prey species (s=1,2,3,4) and a is age of prey.

The rightmost column of Table E 6.1 is the sum

$$\sum_b \overline{N}(1973,1,b) FOOD(1,b) \frac{\overline{N}(1973,s,a)}{\sum_i \sum_d \overline{N}(1973,i,d) SUIT(i,d,1,b)\ \overline{w}(i,d)}$$

i.e. the total number of prey s age a devoured by cod during year 1973. The row "tot.(biom.)" is

$$\sum_s \sum_a \overline{N}(1973,1,b) FOOD(1,b)\ \frac{\overline{N}(1973,s,a)\ \overline{w}(s,a)}{\sum_i \sum_d \overline{N}(1973,i,d) SUIT(i,d,1,b)\ \overline{w}(i,d)}$$

i.e. the total biomass of food consumed by cod (age group b) during year 1973.

The last row "avail. food" is

$$\sum_i \sum_d \overline{N}(1973,i,d) SUIT(i,d,1,b)\ \overline{w}(i,d)$$

i.e. the biomass of food available to cod agegroup b in year 1973.

Table E 6.2 shows the relative contents of cod stomachs. E.g. 2.0631 percent of the five year old cod's stomach content was 2 years old plaice in 1973. The 1 group cod's diet does not include any of the considered fish species, a result which follows from Table E 3. Notice that the column sum is 1.0. The last row is FOOD(1,b), the value of which was given also in Table E 2.

Table E 7.1 contains the usual output tables of VPA for cod, i.e. catch in numbers, fishing mortalities and stock numbers in the beginning of the year. These tables are assumed to be well known by the reader. The two last tables present the number dead due to predation i.e. D(y,s,a) and predation mortalities.

Tables E.7.2-3 present the VPA tables of herring and plaice.

Table E 8 contains the average mortalities over a number of years specified by the user, e.g. (F(73,2,1)+F(74,2,1)+F(75,2,1))/3 = .73. Table E 8 brings us to the end of VPA and the remaining tables deal with the prognosis.

Table E 9 does not need to be explained further. Table E 10 specifies the characteristica of the two fleets considered. The two fleets of this hypothetical example are named "consump" and "industr". The selection curve of the gear are given by the selection factor (SEL(s,e)), the mesh size and (L75%/L50%), from which the program calculates

L50% = (mesh size) X (selection factor) and
L75% = L50% X (L75%/L50%)

Cod is the target species of the consumption fleet. It is seen that a certain F exerted on cod will produce a fishing mortality of 0.4 F on plaice and no F on herring (recall that this is a hypothetical example). The discard curve is determined from LD50% and (LD75%/LD50%).

For the industrial fleet LD50% is given the value 1.0 which cause the program to give FDISC the value zero. The choice of selection factor and mesh size for the industrial fleet secure that no fish escape through the meshes of the industrial trawl.

Recruitment to fishing grounds was ignored in this version of the program.

Table E 11 shows the EF(e,y) values. Table E 12 shows F(e,y,s,a) and Table E 13 presents (F(y,s,a). Notice that the values of F(1978,s,a) in Table E 13 differ slightly from those in Tables E 7.1-3. Usually the user are expected to choose the gear selection parameters so that the two F(1978,s,a)-arrays do not differ markedly.

Table E 14 contains FLAND(e,y,s,a) and FDISC(e,y,s,a).

Table E 15 shows the stock/recruitment parameters.

In this version of the program the parameter SPAW (cf. section 5.2) is defined by

SPAW = (fecundity) X W X 0.5

The first column of Table E 15 is the fecundity (= 2 SPAW/W).

Table E 16 shows the coefficients V(e,y,s,a)of the goal function and the rate of interest. In this case the rate of interest is zero and all Vs are 1.0. This choice of V and r implies that the goal function equals the total biomass landed by both fisheries.

Table E 17 shows the numbers in the sea at the beginning of the starting year. These figures are calculated from the VPA Tables E 7.1-3 by N(1979,s,a) = N(1978,s,a-1)exp(-Z(1978,s,a-1)) where Z(1978,s,a-1) are those of Table E 13.

Table E 18 and E 19.1-4 correspond to Tables E 5 and E 6.1-2 of VPA. Tables E 20.1-3 present the prognoses for the years 1979-81 with reference to the fish stocks. Tables E 21.1-3 give prognoses with reference to the consumption fleet. The Table "goal function" contains the values of

$$\text{YIELD}(1,y,s,a)\ V(1,y,s,a)(1 + r)^{-(y-1979)}$$

The figures of Table 21.4 are

$$\sum_{a} \text{YIELD}(1,y,s,a)\ V(1,y,s,a)\ (1+r)^{-(y-1979)}$$

the values of which are also given in Tables E 21.1-3. The figure

$$1133014 = \sum_{y=1979}^{1981} \sum_{s=1}^{3} \sum_{a} \mathrm{YIELD}(1,y,s,a)\ V(1,y,s,a)(1+r)^{-(y-1979)}$$

is the total return from the consumption fishery.

Tables E 22.5-8 show the similar tables for the industrial fleet. The sum

$$1133014 + 34764 = 1167778 =$$

$$\sum_{e=1}^{2} \sum_{y=1979}^{1981} \sum_{s=1}^{3} \sum_{a} \mathrm{YIELD}(e,y,s,a)\ 1.0\ (1+0)^{-(y-1979)}$$

is the value of the goal function.

Table E1. INPUT.

multispecies cohort analysis

---

number of species : 4
first year (YFIRST) : 1973 last year (YLAST) : 1978

		youngest YAGE(s)	oldest OAGE(s)	spaw.age MAGE(s)
1	cod.......	1	7	3
2	herring...	1	5	3
3	plaice....	1	7	4
4	other.....	1	1	1

fish predators :

1 cod.......

total biomass of the ecosystem : 8000000

Table E2. INPUT.

	species s	age a	weight W(s,a)	tot. consump. FOOD(s,a)	other m M1(a)
1	cod.......	1	0.500	2.975	0.100
2		2	0.900	4.384	0.100
3		3	2.020	7.475	0.100
4		4	3.830	11.403	0.100
5		5	5.730	14.876	0.100
6		6	7.750	18.157	0.100
7		7	9.130	20.231	0.100
8	herring...	1	0.090	0.247	0.100
9		2	0.121	0.300	0.100
10		3	0.158	0.358	0.100
11		4	0.175	0.383	0.100
12		5	0.186	0.399	0.100
13	plaice....	1	0.110	0.506	0.100
14		2	0.225	0.811	0.100
15		3	0.338	1.061	0.100
16		4	0.450	1.281	0.100
17		5	0.563	1.485	0.100
18		6	0.664	1.656	0.100
19		7	0.750	1.795	0.100

Table E3. INPUT.

food suitability matrix. SUIT(s,a,j,b) :

predator : cod.......		age	1	2	3	4	5	6	7
prey: cod.......	age:	1	-	-	-	0.018	0.024	0.030	0.034
		2	-	-	-	-	-	0.011	0.013
prey: herring...	age:	1	-	0.295	0.253	0.203	0.171	0.150	0.139
		2	-	0.168	0.169	0.154	0.140	0.130	0.125
		3	-	-	0.112	0.115	0.112	0.110	0.108
		4	-	-	0.094	0.101	0.101	0.101	0.101
		5	-	-	0.085	0.093	0.095	0.096	0.097
prey: plaice....	age:	1	-	0.203	0.194	0.170	0.150	0.137	0.130
		2	-	-	0.060	0.072	0.077	0.081	0.083
		3	-	-	-	0.038	0.045	0.052	0.055
		4	-	-	-	0.022	0.029	0.035	0.039
		5	-	-	-	-	0.020	0.025	0.028
		6	-	-	-	-	0.015	0.019	0.022
		7	-	-	-	-	0.012	0.015	0.018
prey: other.....	age:	1	1.000	0.334	0.033	0.014	0.009	0.007	0.006

Table E4. INPUT.

initial guess on F and M2 for oldest age group
F(y,s,OAGE(s)) and M2(y,s,OAGE(s))

year	species	1	2	3
1973	F:	0.700	0.500	0.300
	M2:	0.000	0.100	0.000
1974	F:	0.700	0.500	0.300
	M2:	0.000	0.100	0.000
1975	F:	0.700	0.500	0.300
	M2:	0.000	0.100	0.000
1976	F:	0.700	0.500	0.300
	M2:	0.000	0.100	0.000
1977	F:	0.700	0.400	0.300
	M2:	0.000	0.100	0.000
1978	F:	0.680	0.100	0.280
	M2:	0.000	0.100	0.000

initial guess on F and M2 for last year
F(YLAST,s,a) and M2(YLAST,s,a)

cod........

	age	F	M2
1	1	0.200	0.200
1	2	0.680	0.200
1	3	0.680	0.100
1	4	0.680	0.000
1	5	0.680	0.000
1	6	0.680	0.000
1	7	0.680	0.000

herring...

	age	F	M2
2	1	0.100	0.400
2	2	0.100	0.300
2	3	0.100	0.200
2	4	0.100	0.100
2	5	0.100	0.100

plaice....

	age	F	M2
3	1	0.020	0.300
3	2	0.100	0.200
3	3	0.240	0.100
3	4	0.280	0.100
3	5	0.280	0.000
3	6	0.280	0.000
3	7	0.280	0.000

Table E5. OUTPUT.

1977	no. of iterations :	4	tot. fish biom.	1073603
1976	no. of iterations :	4	tot. fish biom.	1001413
1975	no. of iterations :	4	tot. fish biom.	1428257
1974	no. of iterations :	4	tot. fish biom.	1241460
1973	no. of iterations :	4	tot. fish biom.	1692636

cpu time of vpa iteratins : 3.01 sec.

Table E6.1. OUTPUT.

w h o   e a t s   w h o   (in numbers) matrix for year : 1973

predator : cod.......

	age	1	2	3	4	5	6	7		total
cod.......	1	0	0	0	1603	677	454	232	!	2966
	2	0	0	0	0	0	52	28	!	81
herring...	1	0	62404	535894	497922	132613	63024	26664	!	1318520
	2	0	17583	177109	186702	53839	27158	11867	!	474258
	3	0	0	32531	38427	11904	6336	2851	!	92049
	4	0	0	939	1158	369	200	91	!	2757
	5	0	0	468	592	192	105	48	!	1406
plaice....	1	0	9575	91614	92729	26064	12898	5578	!	238457
	2	0	0	7717	10599	3610	2063	965	!	24954
	3	0	0	0	4459	1694	1051	514	!	7718
	4	0	0	0	2418	992	652	329	!	4391
	5	0	0	0	0	282	194	100	!	577
	6	0	0	0	0	54	38	20	!	113
	7	0	0	0	0	25	18	10	!	54
other.....	1	330246	143433	143153	68115	13692	5657	2275	!	706570
tot.(biom.)		330246	152230	230019	157885	39373	18625	7957	!	936335
avail.food		6307364	2233430	336471	200095	157321	137124	128631		

Table E6.2. OUTPUT.

s t o m a c h   c o n t e n t s   matrix, STOC(s,a,j,b), for year :1973

predator : cod.......

	age	1	2	3	4	5	6	7
cod.......	1	0.000000	0.000000	0.000000	0.005077	0.008593	0.012193	0.014562
	2	0.000000	0.000000	0.000000	0.000000	0.000000	0.002524	0.003214
herring...	1	0.000000	0.036894	0.209680	0.283834	0.303129	0.304539	0.301570
	2	0.000000	0.013976	0.093167	0.143085	0.165455	0.176430	0.180452
	3	0.000000	0.000000	0.022346	0.038455	0.047770	0.053750	0.056601
	4	0.000000	0.000000	0.000714	0.001284	0.001639	0.001883	0.002005
	5	0.000000	0.000000	0.000378	0.000698	0.000906	0.001053	0.001129
plaice....	1	0.000000	0.006919	0.043812	0.064605	0.072818	0.076172	0.077102
	2	0.000000	0.000000	0.007548	0.015104	0.020631	0.024926	0.027282
	3	0.000000	0.000000	0.000000	0.009546	0.014545	0.019073	0.021824
	4	0.000000	0.000000	0.000000	0.006892	0.011341	0.015753	0.018598
	5	0.000000	0.000000	0.000000	0.000000	0.004039	0.005869	0.007101
	6	0.000000	0.000000	0.000000	0.000000	0.000911	0.001368	0.001686
	7	0.000000	0.000000	0.000000	0.000000	0.000484	0.000744	0.000929
other.....	1	1.000000	0.942211	0.622354	0.431421	0.347741	0.303722	0.285945
tot. consum.		2.9745	4.3843	7.4753	11.4027	14.8758	18.1567	20.2306

Table E7.1. INPUT: Numbers caught, final Fs and final M2s (indicated by ★). The M2s of last year, 1978, are given the same value as those of 1977.
OUTPUT: F, M2 (except for the final ones), stock numbers and numbers of deaths due to predation.

v. p. a.   r e s u l t s :

legion analysis

numbers caught of cod....... C(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	34000	16000	35000	7000	109000	39000
2	31000	53000	55000	87000	52000	169000
3	32000	12000	18000	19000	22600	15000
4	13000	5000	4000	6000	4000	8000
5	2000	4000	6000	1400	1700	2500
6	1000	1000	2000	2200	600	800
7	500	500	500	1000	1000	300
biomass	183105	134325	173125	177362	185793	255804

the last group is a plus group

fishing mortality of cod....... F(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.3062	0.1504	0.2122	0.0692	0.2318	0.2000*
2	0.8928	0.9768	0.9839	1.0747	0.9067	0.6800*
3	1.0400	0.9607	0.9746	1.0243	0.8153	0.6800*
4	0.9389	0.3821	0.9032	0.9367	0.5395	0.6800*
5	0.7556	0.7556	0.9525	0.8391	0.6672	0.6800*
6	0.9748	0.9748	0.9748	1.0349	0.9726	0.6800*
7	0.7000*	0.7000*	0.7000*	0.7000*	0.7000*	0.6800*

average f (weighted ny stock numbers (3<= age <= 7))

	1973	1974	1975	1976	1977	1978
	0.9941	0.7200	0.9574	0.9890	0.7588	0.6800

stock in numbers of cod....... N(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	136788	122544	195114	111940	560907	247796
2	54817	88719	91729	138030	91138	389888
3	51571	20265	30137	30891	42316	33170
4	22262	16494	7016	10290	10036	16943
5	3939	7877	10185	2573	3649	5295
6	1674	1674	3348	3555	1006	1694
7	571	571	571	1143	1143	344
biomass	347926	308554	357385	334737	525533	653307
spawning stock (age > 2) SSB(y,s) biomass	230197	167435	177272	154540	163055	178510

numbers dead due to predation on cod....... D(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	2966	4175	5590	3683	14994	10159
2	81	157	248	619	233	1106
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0
biomass	1555	2229	3018	2399	7707	6075

predation mortality of cod....... M2(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.0267	0.0392	0.0339	0.0364	0.0319	0.0319*
2	0.0023	0.0029	0.0044	0.0077	0.0041	0.0041*
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000*
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000*
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000*
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000*
7	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*	0.0000*

Table E7.2. INPUT: Numbers caught, final Fs and final M2s (indicated by ★). The M2s of last year, 1978, are given the same value as those of 1977.
OUTPUT: F, M2 (except for the final ones), stock numbers and numbers of deaths due to predation.

v. p. a. r e s u l t s :

legion analysis

numbers caught of herring... C(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	2368000	846000	2461000	127000	144000	13700
2	1344000	773000	542000	902000	45000	4000
3	659000	362000	260000	117000	98000	5000
4	15000	126000	141000	52000	7000	5000
5	8000	5000	4200	45000	9000	1000
biomass	483979	249849	353608	156528	36788	3568

the last group is a plus group

fishing mortality of herring... F(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.7632	0.5567	0.8688	0.4898	1.0955	0.1000*
2	0.8769	0.9250	1.3656	1.5173	0.4562	0.1000*
3	1.5498	0.8054	1.3431	2.1776	0.8695	0.1000*
4	1.0309	2.8993	1.0626	1.4355	1.0762	0.1000*
5	0.5000*	0.5000*	0.5000*	0.5000*	0.4000*	0.1000*
average f (weighted ny stock numbers (3<= age <= 5))	1.5239	1.1390	1.2300	1.6092	0.8520	0.1000

stock in numbers of herring... N(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	5514086	2493137	5149612	412175	270644	182186
2	2723296	1523800	840277	1343418	149893	50830
3	938718	753002	394537	146195	194476	60665
4	26209	145244	241070	76187	12037	57874
5	11200	7000	5880	63000	13500	3000
biomass	980774	554456	670756	247799	77840	42818
spawning stock (age > 2) SSB(y,s) biomass	154987	145694	105618	48150	35345	20271

numbers dead due to predation on herring... D(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	2630544	1312959	2127361	219254	125592	96704
2	947285	545954	224951	375268	68833	34099
3	184002	210074	78009	23578	54691	27796
4	5512	18040	47615	14133	2774	23683
5	2811	1941	1412	16440	3998	1144
biomass	381930	220936	239602	74397	29502	21579

predation mortality of herring... M2(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.4229	0.4309	0.3749	0.4217	0.4768	0.4768*
2	0.3086	0.3263	0.2832	0.3154	0.3484	0.3484*
3	0.2163	0.2336	0.2014	0.2194	0.2425	0.2425*
4	0.1893	0.2075	0.1794	0.1950	0.2131	0.2131*
5	0.1745*	0.1928*	0.1670*	0.1814*	0.1969*	0.1969*

Table E7.3. INPUT: Numbers caught, final Fs and final M2s (indicated by ★). The M2s of last year, 1978, are given the same value as those of 1977.
OUTPUT: F, M2 (except for the final ones), stock numbers and numbers of deaths due to predation.

v. p. a. r e s u l t s :

legion analysis

numbers caught of plaice.... C(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	1500	1500	500	4000	2100	600
2	25000	20000	36000	29000	53000	53000
3	66000	60000	92000	72000	54000	48000
4	82000	48000	38000	105000	63000	45000
5	40000	41000	27000	17000	52000	25000
6	15000	21000	26000	13000	8000	16000
7	8000	6000	10000	11000	4000	2000
biomass	103478	88072	96316	105004	96346	74664

the last group is a plus group

fishing mortality of plaice.... F(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.0022	0.0028	0.0010	0.0084	0.0025	0.0200*
2	0.1334	0.0420	0.1003	0.0830	0.1722	0.1000*
3	0.4402	0.5417	0.2773	0.2974	0.2195	0.2400*
4	0.6007	0.6275	0.7609	0.5511	0.4348	0.2800*
5	0.7015	0.6292	0.8253	0.8715	0.5366	0.2800*
6	1.0153	0.9060	0.9764	1.1855	1.3221	0.2800*
7	0.3000*	0.3000*	0.3000*	0.3000*	0.3000*	0.2800*
average f (weighted ny stock numbers (3<= age <= 7))	0.5684	0.6139	0.4379	0.4591	0.3705	0.2643

stock in numbers of plaice.... N(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	857519	668507	618032	595453	1048616	36740
2	223825	549521	421647	409409	377589	643048
3	198623	155153	410042	302566	295629	247216
4	192576	109912	75781	264259	190328	202360
5	83255	92534	50656	30775	131985	107322
6	24590	36975	43800	19613	11395	68686
7	10667	8000	13333	14667	5333	2714
biomass	369682	381728	403152	420150	471749	431413
spawning stock (age > 3) SSB(y,s) biomass	157859	132108	101704	160266	171521	199127

numbers dead due to predation on plaice.... D(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	475649	384482	315538	332846	642836	16866
2	49887	143725	94422	99704	93210	193408
3	15438	16591	41198	32051	29327	30896
4	8784	7213	4023	16443	11035	15872
5	1154	2435	1541	858	3214	3425
6	225	648	941	367	152	1652
7	108	147	310	327	89	52
biomass	73597	85397	73414	78251	108539	66022

predation mortality of plaice.... M2(y,s,a)

age/year	1973	1974	1975	1976	1977	1978
1	0.3427	0.3580	0.3108	0.3472	0.3865	0.3865*
2	0.1330	0.1508	0.1315	0.1426	0.1514	0.1514*
3	0.0515	0.0749	0.0621	0.0662	0.0596	0.0596*
4	0.0322	0.0471	0.0403	0.0432	0.0381	0.0381*
5	0.0101	0.0187	0.0236	0.0220	0.0166	0.0166*
6	0.0076	0.0140	0.0177	0.0167	0.0125	0.0125*
7	0.0061*	0.0112*	0.0141*	0.0135*	0.0101*	0.0101*

Table E8. OUTPUT.

average fishing mortalities (over years)

age		0	1	2	3	4	5	6	7
cod.......									
years :	1973 1975		0.22	0.95	0.99	0.74	0.82	0.97	0.70
herring...									
years :	1973 1975		0.73	1.06	1.23	1.66	0.50		
plaice....									
years :	1973 1975		0.00	0.09	0.42	0.66	0.72	0.97	0.30

Table E9. INPUT.

input for prognosis calculations

prognosis for the years : 1979 1981 (LASTY+1 and FORY)

number of fleets : 2 (E)

bertalanffy parameters :

		l8	k	to
cod........	1	130.000	0.300	0.800
herring...	2	35.000	0.300	-1.000
plaice....	3	38.000	0.100	-0.800

Table E10. INPUT.

fleet : consump...

mesh size : 9.00 cm MESH(e)

species no.	1	2	3	
selection factor	3.00	2.00	1.30	SEL(s,e)
l75/l50	1.10	1.10	1.10	LL(s,e)
distribution of bycatches	1.00	0.00	0.40	BYC(e,s)
discard l50	30.00	1.00	10.00	LD50o/o(s,e)
discard l75/l50	1.10	1.10	1.10	
right gear selec.	130.0	35.0	38.0	RL50o/o(s,e)
right l75/l50	1.30	1.30	1.30	

fleet : industr...

mesh size : 1.00 cm MESH(e)

species no.	1	2	3	
selection factor	1.00	1.00	1.00	SEL(s,e)
l75/l50	1.10	1.10	1.10	LL(s,e)
distribution of bycatches	0.10	1.00	0.20	BYC(e,s)
discard l50	1.00	1.00	1.00	LD50o/o(s,e)
discard l75/l50	1.10	1.10	1.10	
right gear selec.	50.0	35.0	20.0	RL50o/o(s,e)
right l75/l50	1.30	1.30	1.30	

Table E11. INPUT.

max fishing mortalities EF(e,y)

fleet no.	1	2
year		
1978	0.68	0.10
1979	0.68	0.10
1980	0.68	0.10
1981	0.68	0.10

Table E12. OUTPUT.

fleet : consump...

fishing mortality from consump... F(e,y,s,a)

cod.......	1978	1979	1980	1981
1	0.19	0.19	0.19	0.19
2	0.68	0.68	0.68	0.68
3	0.68	0.68	0.68	0.68
4	0.68	0.68	0.68	0.68
5	0.68	0.68	0.68	0.68
6	0.68	0.68	0.68	0.68
7	0.68	0.68	0.68	0.68

fishing mortality from consump... F(e,y,s,a)

plaice....	1978	1979	1980	1981
1	0.01	0.01	0.01	0.01
2	0.08	0.08	0.08	0.08
3	0.22	0.22	0.22	0.22
4	0.27	0.27	0.27	0.27
5	0.27	0.27	0.27	0.27
6	0.27	0.27	0.27	0.27
7	0.27	0.27	0.27	0.27

fleet : industr...

fishing mortality from industr... F(e,y,s,a)

cod.......	1978	1979	1980	1981
1	0.01	0.01	0.01	0.01
2	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00

fishing mortality from industr... F(e,y,s,a)

herring...	1978	1979	1980	1981
1	0.10	0.10	0.10	0.10
2	0.10	0.10	0.10	0.10
3	0.10	0.10	0.10	0.10
4	0.10	0.10	0.10	0.10
5	0.10	0.10	0.10	0.10

fishing mortality from industr... F(e,y,s,a)

plaice....	1978	1979	1980	1981
1	0.02	0.02	0.02	0.02
2	0.02	0.02	0.02	0.02
3	0.02	0.02	0.02	0.02
4	0.01	0.01	0.01	0.01
5	0.01	0.01	0.01	0.01
6	0.01	0.01	0.01	0.01
7	0.01	0.01	0.01	0.01

Table E13. OUTPUT.

fishing mortality on each species :

F for : cod....... F(y,s,a)

age/year	1978	1979	1980	1981
1	0.1960	0.1960	0.1960	0.1960
2	0.6846	0.6846	0.6846	0.6846
3	0.6816	0.6816	0.6816	0.6816
4	0.6806	0.6806	0.6806	0.6806
5	0.6803	0.6803	0.6803	0.6803
6	0.6802	0.6802	0.6802	0.6802
7	0.6801	0.6801	0.6801	0.6801

F for : herring... F(y,s,a)

age/year	1978	1979	1980	1981
1	0.1000	0.1000	0.1000	0.1000
2	0.1000	0.1000	0.1000	0.1000
3	0.1000	0.1000	0.1000	0.1000
4	0.1000	0.1000	0.1000	0.1000
5	0.1000	0.1000	0.1000	0.1000

F for : plaice.... F(y,s,a)

age/year	1978	1979	1980	1981
1	0.0249	0.0249	0.0249	0.0249
2	0.0924	0.0924	0.0924	0.0924
3	0.2372	0.2372	0.2372	0.2372
4	0.2792	0.2792	0.2792	0.2792
5	0.2831	0.2831	0.2831	0.2831
6	0.2821	0.2821	0.2821	0.2821
7	0.2807	0.2807	0.2807	0.2807

Table E14. OUTPUT.

landing- and discard mortality of each fleet

fleet : consump...

F-land. and F-disc. from consump...
FLAND(e,y,s,a) and FDISC(e,y,s,a)

cod.......			1978	1979	1980	1981
land.	F.	1	0.02	0.02	0.02	0.02
disc.	F.		0.16	0.16	0.16	0.16
land.	F.	2	0.68	0.68	0.68	0.68
land.	F.	3	0.68	0.68	0.68	0.68
land.	F.	4	0.68	0.68	0.68	0.68
land.	F.	5	0.68	0.68	0.68	0.68
land.	F.	6	0.68	0.68	0.68	0.68
land.	F.	7	0.68	0.68	0.68	0.68

F-land. and F-disc. from consump...
FLAND(e,y,s,a) and FDISC(e,y,s,a)

plaice....			1978	1979	1980	1981
land.	F.	1	0.00	0.00	0.00	0.00
disc.	F.		0.01	0.01	0.01	0.01
land.	F.	2	0.05	0.05	0.05	0.05
disc.	F.		0.02	0.02	0.02	0.02
land.	F.	3	0.22	0.22	0.22	0.22
disc.	F.		0.01	0.01	0.01	0.01
land.	F.	4	0.26	0.26	0.26	0.26
disc.	F.		0.00	0.00	0.00	0.00
land.	F.	5	0.27	0.27	0.27	0.27
land.	F.	6	0.27	0.27	0.27	0.27
land.	F.	7	0.27	0.27	0.27	0.27

(cont..)

Table E14. (cont.) OUTPUT.

fleet : industr...

F-land. and F-disc. from industr...
FLAND(e,y,s,a) and FDISC(e,y,s,a)

cod.......			1978	1979	1980	1981
land.	F.	1	0.01	0.01	0.01	0.01
land.	F.	2	0.00	0.00	0.00	0.00
land.	F.	3	0.00	0.00	0.00	0.00
land.	F.	4	0.00	0.00	0.00	0.00
land.	F.	5	0.00	0.00	0.00	0.00
land.	F.	6	0.00	0.00	0.00	0.00
land.	F.	7	0.00	0.00	0.00	0.00

F-land. and F-disc. from industr...
FLAND(e,y,s,a) and FDISC(e,y,s,a)

herring...			1978	1979	1980	1981
land.	F.	1	0.10	0.10	0.10	0.10
land.	F.	2	0.10	0.10	0.10	0.10
land.	F.	3	0.10	0.10	0.10	0.10
land.	F.	4	0.10	0.10	0.10	0.10
land.	F.	5	0.10	0.10	0.10	0.10

F-land. and F-disc. from industr...
FLAND(e,y,s,a) and FDISC(e,y,s,a)

plaice....			1978	1979	1980	1981
land.	F.	1	0.02	0.02	0.02	0.02
land.	F.	2	0.02	0.02	0.02	0.02
land.	F.	3	0.02	0.02	0.02	0.02
land.	F.	4	0.01	0.01	0.01	0.01
land.	F.	5	0.01	0.01	0.01	0.01
land.	F.	6	0.01	0.01	0.01	0.01
land.	F.	7	0.01	0.01	0.01	0.01

Table E15. INPUT.

parameters in stock/recruitment model :

	2*SPAW/W	NOMAX	1/HALFSAT
cod.......	0.30000	200000	0.00100
herring...	0.30000	1000000	0.00100
plaice....	0.30000	400000	0.00100

Table E16. INPUT.

goal function :
rate of interest 0.0000 (r)

weights in the goal function of fleet :consump... V(y,e,s,a)

age	0	1	2	3	4	5	6	7
cod.......		1.00	1.00	1.00	1.00	1.00	1.00	1.00
herring...		1.00	1.00	1.00	1.00	1.00		
plaice....		1.00	1.00	1.00	1.00	1.00	1.00	1.00
other.....		1.00						

weights in the goal function of fleet :industr... V(y,e,s,a)

age	0	1	2	3	4	5	6	7
cod.......		1.00	1.00	1.00	1.00	1.00	1.00	1.00
herring...		1.00	1.00	1.00	1.00	1.00		
plaice....		1.00	1.00	1.00	1.00	1.00	1.00	1.00
other.....		1.00						

Table E17. OUTPUT, except for the 0-group.

numbers at the beginning of starting year 1979
stock in numbers

age	1	2	3	4	5	6	7	biom.	SSB(y,s)
cod.......	200000	143513	178003	15205	7767	2427	934	718811	489649
herring...	1000000	508247	29374	38973	40305			170456	18958
plaice....	400000	241046	452532	165781	133214	72186	48224	484892	233701
							total :	1374159	742308

Table E18. OUTPUT.

1979	no. of iterations :	3	fish biomass :	1348180
1980	no. of iterations :	4	fish biomass :	1342482
1981	no. of iterations :	4	fish biomass :	1179642

cpu time for prognosis iterations : 0.95 sec.

Table E19.1. OUTPUT.

w h o   e a t s   w h o   (in numbers) matrix for year : 1980

predator : cod....... age		1	2	3	4	5	6	7	!	total
cod.......	1	0	0	0	16890	3491	3165	4719	!	28265
	2	0	0	0	0	0	704	1119	!	1822
herring...	1	0	18392	120342	399096	52053	33410	41301	!	664594
	2	0	2112	16206	60977	8611	5866	7490	!	101263
	3	0	0	13188	55599	8435	6063	7971	!	91256
	4	0	0	938	4129	644	472	628	!	6812
	5	0	0	2250	10168	1612	1197	1602	!	16829
plaice....	1	0	6933	50545	182603	25135	16798	21226	!	303240
	2	0	0	6082	29815	4974	3839	5245	!	49955
	3	0	0	0	20960	3900	3268	4668	!	32795
	4	0	0	0	28724	5772	5123	7550	!	47168
	5	0	0	0	0	1542	1431	2162	!	5134
	6	0	0	0	0	952	913	1405	!	3270
	7	0	0	0	0	1033	1016	1584	!	3633
other.....	1	467699	414134	314938	534866	52651	29380	34529	!	0
tot.(biom.)		467699	416807	337324	644812	71943	44871	56014	!	2039470
avail.food		6657518	2235525	236740	109847	78902	67139	62979		

Table E19.2. OUTPUT.

w h o   e a t s   w h o   (in numbers) matrix for year : 1981

predator : cod.......

	age	1	2	3	4	5	6	7	total
cod.......	1	0	0	0	6184	19160	2971	6290 !	34605
	2	0	0	0	0	0	598	1350 !	1948
herring...	1	0	14659	104158	137451	268687	29506	51790 !	606251
	2	0	1187	9891	14808	31342	3653	6623 !	67503
	3	0	0	2076	3483	7919	974	1818 !	16271
	4	0	0	2896	5075	11859	1489	2810 !	24129
	5	0	0	3359	6041	14350	1824	3465 !	29038
plaice....	1	0	5922	46884	67399	139045	15899	28525 !	303674
	2	0	0	3406	6645	16613	2194	4256 !	33114
	3	0	0	0	2546	7102	1018	2065 !	12731
	4	0	0	0	2396	7217	1096	2293 !	13002
	5	0	0	0	0	12493	1984	4256 !	18733
	6	0	0	0	0	3995	656	1433 !	6084
	7	0	0	0	0	6911	1163	2573 !	10647
other.....	1	457887	367196	303238	204926	302333	28864	48167 !	0
tot.(biom.)		457887	369310	321193	235590	385433	40243	70516 !	1880172
avail.food		6820358	2288619	239842	107314	75416	62790	58227	

Table E19.3. OUTPUT.

s t o m a c h   c o n t e n t s   matrix, STOC(s,a,j,b), for year :1980

predator : cod.......	age	1	2	3	4	5	6	7
cod.......	1	0.000000	0.000000	0.000000	0.013097	0.024266	0.035267	0.042120
	2	0.000000	0.000000	0.000000	0.000000	0.000000	0.014114	0.017975
herring...	1	0.000000	0.003971	0.032108	0.055704	0.065118	0.067013	0.066361
	2	0.000000	0.000613	0.005813	0.011443	0.014483	0.015819	0.016180
	3	0.000000	0.000000	0.006177	0.013624	0.018525	0.021351	0.022484
	4	0.000000	0.000000	0.000486	0.001121	0.001566	0.001843	0.001962
	5	0.000000	0.000000	0.001240	0.002933	0.004167	0.004963	0.005320
plaice....	1	0.000000	0.001830	0.016482	0.031151	0.038432	0.041180	0.041684
	2	0.000000	0.000000	0.004057	0.010404	0.015555	0.019250	0.021070
	3	0.000000	0.000000	0.000000	0.010987	0.018324	0.024613	0.028165
	4	0.000000	0.000000	0.000000	0.020046	0.036104	0.051373	0.060[illegible]3
	5	0.000000	0.000000	0.000000	0.000000	0.012063	0.017957	0.021[illegible]6
	6	0.000000	0.000000	0.000000	0.000000	0.008784	0.013517	0.016652
	7	0.000000	0.000000	0.000000	0.000000	0.010770	0.016984	0.021203
other.....	1	1.000000	0.993586	0.933636	0.829492	0.731843	0.654755	0.616445
tot. consum.		2.9745	4.3843	7.4753	11.4027	14.8758	18.1567	20.2306

Table E19.4. OUTPUT.

s t o m a c h   c o n t e n t s   matrix, STOC(s,a,j,b), for year :1981

predator : cod.......	age	1	2	3	4	5	6	7
cod.......	1	0.000000	0.000000	0.000000	0.013125	0.024855	0.036919	0.044602
	2	0.000000	0.000000	0.000000	0.000000	0.000000	0.013372	0.017226
herring...	1	0.000000	0.003572	0.029186	0.052509	0.062739	0.065986	0.066101
	2	0.000000	0.000389	0.003726	0.007606	0.009839	0.010984	0.011364
	3	0.000000	0.000000	0.001021	0.002336	0.003246	0.003824	0.004074
	4	0.000000	0.000000	0.001578	0.003770	0.005385	0.006475	0.006974
	5	0.000000	0.000000	0.001945	0.004769	0.006925	0.008429	0.009141
plaice....	1	0.000000	0.001764	0.016057	0.031469	0.039683	0.043457	0.044497
	2	0.000000	0.000000	0.002386	0.006346	0.009698	0.012266	0.013581
	3	0.000000	0.000000	0.000000	0.003653	0.006228	0.008550	0.009897
	4	0.000000	0.000000	0.000000	0.004577	0.008426	0.012253	0.014634
	5	0.000000	0.000000	0.000000	0.000000	0.018248	0.027761	0.033977
	6	0.000000	0.000000	0.000000	0.000000	0.006883	0.010825	0.013489
	7	0.000000	0.000000	0.000000	0.000000	0.013447	0.021674	0.027370
other.....	1	1.000000	0.994275	0.944101	0.869840	0.784398	0.717226	0.683073
tot. consum.		2.9745	4.3843	7.4753	11.4027	14.8758	18.1567	20.2306

Table E20.2. OUTPUT.

p r o g n o s i s   f o r   y e a r : 1980

multispecies model
catch in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	30821	65086	30759	38488	3290	1681	1883	332602
herring...	33437	6730	8271	697	1859	-	-	5598
plaice....	4567	6549	22472	60852	24181	20162	27563	84629
						total :		422830

stock in numbers

age	1	2	3	4	5	6	7	biom.	SSB(y,s)
cod.......	197593	138340	65035	81463	6966	3559	3988	770587	547284
herring...	823803	140205	147985	11863	30710			122277	31170
plaice....	392266	107348	130754	289260	105735	87836	120137	449619	338121
						total :		1342482	916575

number of deaths due to predation

age	1	2	3	4	5	6	7	biom.
cod.......	28265	1822	0	0	0	0	0	15773
herring...	1329345	202546	182527	13624	33659			181633
plaice....	606545	99916	65591	94337	10268	6540	7265	169395
						total :		366801

predation mortality

age	1	2	3	4	5	6	7
cod.......	0.180	0.019	0.000	0.000	0.000	0.000	0.000
herring...	1.988	1.505	1.103	0.977	0.906		
plaice....	1.655	0.705	0.346	0.216	0.060	0.046	0.037

Table E20.1. OUTPUT.

p r o g n o s i s   f o r   y e a r : 1979

multispecies model

catch in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	32558	67882	84189	7184	3668	1146	441	308879
herring...	36093	29226	1933	2670	2825			8083
plaice....	5457	16670	86559	37281	30848	16730	11152	87224
							total :	404186

stock in numbers

age	1	2	3	4	5	6	7	biom.	SSB(y,s)
cod.......	197314	143513	178003	15205	7767	2427	934	717467	489649
herring...	739836	508247	29374	38973	40305			147041	18958
plaice....	388906	241046	452532	165781	133214	72186	48224	483672	233701
							total :	1348180	742308

number of deaths due to predation

age	1	2	3	4	5	6	7	biom.
cod.......	9812	682	0	0	0	0	0	5520
herring...	1055747	604059	27309	32678	31636			184026
plaice....	508767	151245	80511	18840	7276	2976	1597	132955
							total :	322501

predation mortality

age	1	2	3	4	5	6	7
cod.......	0.059	0.007	0.000	0.000	0.000	0.000	0.000
herring...	1.463	1.034	0.707	0.612	0.560		
plaice....	1.162	0.419	0.110	0.071	0.033	0.025	0.020

Table E20.3. OUTPUT.

prognosis for year: 1981

multispecies model
catch in numbers

age	1	2	3	4	5	6	7	biom.
cod........	30174	57669	29288	14062	17626	1508	2371	314335
herring...	30792	4370	1386	2292	2952			4469
plaice....	4507	3903	7301	13574	34964	15100	32896	64333
						total :		383137

stock in numbers

age	1	2	3	4	5	6	7	biom.	SSB(y,s)
cod........	197485	122786	61925	29764	37320	3192	5020	732753	523503
herring...	753941	92373	25488	40192	50357			99459	20427
plaice....	390526	66149	43760	66017	159449	67879	146999	347430	274798
						total :		1179642	818728

number of deaths due to predation

age	1	2	3	4	5	6	7	biom.
cod........	34605	1948	0	0	0	0	0	19056
herring...	1212645	135020	32544	48262	58080			149866
plaice....	607415	66231	25463	26004	37466	12168	21294	147169
						total :		316091

predation mortality

age	1	2	3	4	5	6	7
cod........	0.225	0.023	0.000	0.000	0.000	0.000	0.000
herring...	1.969	1.545	1.175	1.053	0.984		
plaice....	1.679	0.784	0.414	0.267	0.152	0.114	0.091

Table E21.1. OUTPUT.

prognosis for year : 1979 fleet :consump...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	3813	67399	83985	7177	3667	1146	441	293625
plaice....	124	9242	78768	35366	29532	16120	10806	80066
						total :		373692

discards

age	1	2	3	4	5	6	7	biom.
cod.......	27308	0	0	0	0	0	0	13654
plaice....	1378	4374	2143	73	0	0	0	1893
						total :		15547

goal function

age	1	2	3	4	5	6	7	total
cod.......	1907	60659	169650	27490	21011	8881	4028	293625
plaice....	14	2079	26623	15915	16627	10704	8105	80066
						total :		373692

Table E21.2. OUTPUT.

prognosis for year : 1980 fleet :consump...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	3610	64624	30685	38453	3289	1680	1883	318283
plaice....	104	3631	20450	57726	23150	19426	26709	79682
						total :		397965

discards

age	1	2	3	4	5	6	7	biom.
cod.......	25851	0	0	0	0	0	0	12925
plaice....	1153	1718	556	119	0	0	0	755
						total :		13680

goal function

age	1	2	3	4	5	6	7	total
cod.......	1805	58161	61984	147276	18844	13024	17190	318283
plaice....	11	817	6912	25977	13033	12899	20032	79682
						total :		397965

Table E21.3. OUTPUT.

prognosis for year : 1981 fleet :consump...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	3534	57259	29218	14049	17619	1507	2370	300406
plaice....	102	2164	6643	12877	33473	14549	31877	60952
							total :	361357

discards

age	1	2	3	4	5	6	7	biom.
cod.......	25309	0	0	0	0	0	0	12654
plaice....	1138	1024	181	26	0	0	0	429
							total :	13083

goal function

age	1	2	3	4	5	6	7	total
cod.......	1767	51533	59020	53809	100956	11681	21640	300406
plaice....	11	487	2245	5795	18845	9661	23908	60952
							total :	361357

Table E21.4. OUTPUT.

goal function values for each species and each year fleet :consump...

year	1979	1980	1981	total
cod.......	293625	318283	300406	912314
herring...	0	0	0	0
plaice....	80066	79682	60952	220699
total	373692	397965	361357	1133014

Table E21.5. OUTPUT.

prognosis for year : 1979 fleet :industr...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	1437	461	203	7	2	0	0	1580
herring...	36093	29226	1933	2670	2825			8083
plaice....	3955	3054	5648	1842	1310	610	346	5262
						total :		14925

discards

age	1	2	3	4	5	6	7	biom.
						total :		0

goal function

age	1	2	3	4	5	6	7	total
cod.......	719	415	411	25	9	2	1	1580
herring...	3248	3536	305	467	525			8083
plaice....	435	687	1909	829	737	405	259	5262
						total :		14925

Table E21.6. OUTPUT.

prognosis for year : 1980 fleet :industr...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	1360	442	74	35	1	0	0	1375
herring...	33437	6730	8271	697	1859			5598
plaice....	3310	1200	1466	3007	1027	735	854	4190
						total :		11163

discards

age	1	2	3	4	5	6	7	biom.
						total :		0

goal function

age	1	2	3	4	5	6	7	total
cod.......	680	398	150	134	8	3	3	1375
herring...	3009	814	1307	122	346			5598
plaice....	364	270	496	1353	578	488	641	4190
						total :		11163

Table E21.7. OUTPUT.

prognosis for year : 1981 fleet :industr...

landings in numbers

age	1	2	3	4	5	6	7	biom.
cod.......	1332	391	71	13	7	0	0	1258
herring...	30792	4370	1386	2292	2952			4469
plaice....	3267	715	476	671	1485	551	1019	2949
						total :		8676

discards

age	1	2	3	4	5	6	7	biom.
						total :		0

goal function

age	1	2	3	4	5	6	7	total
cod.......	666	352	143	49	42	3	3	1258
herring...	2771	529	219	401	549			4469
plaice....	359	161	161	302	836	366	765	2949
						total :		8676

Table E21.8. OUTPUT.

goal function values for each species and each year fleet :industr...

year	1979	1980	1981	total
cod.......	1580	1375	1258	4213
herring...	8083	5598	4469	18150
plaice....	5262	4190	2949	12401
total	14925	11163	8676	34764

APPENDIX F.

DERIVATION OF THE FORMULA FOR FOOD SUITABILITY AS A FUNCTION OF STOMACH CONTENTS.

The expression for SUIT as a function of STOC is derived as follows :

$$\mathrm{SUIT}(s,a,j,b) = \mathrm{SUIT}(s,a,j,b,)\cdot 1 =$$

$$\mathrm{SUIT}(s,a,j,b)\,\frac{\dfrac{\bar{N}(y,s,a)\,\bar{w}(s,a)}{\bar{N}(y,s,a)\,\bar{w}(s,a)}}{\sum\limits_i\sum\limits_d \mathrm{SUIT}(i,d,j,b)} =$$

$$\mathrm{SUIT}(s,a,j,b)\,\frac{\dfrac{\bar{N}(y,s,a)\,\bar{w}(s,a)}{\bar{N}(y,s,a)\,\bar{w}(s,a)}}{\sum\limits_i\sum\limits_d \dfrac{\bar{N}(y,i,d)\,\bar{w}(i,d)}{\bar{N}(y,i,d)\,\bar{w}(i,d)}\,\mathrm{SUIT}(i,d,j,b)} =$$

$$\frac{\left(\dfrac{\bar{N}(y,s,a)\,\bar{w}(s,a)\mathrm{SUIT}(s,a,j,b)}{\bar{N}(y,s,a)\,\bar{w}(s,a)}\right)\left(\dfrac{1}{\sum\limits_i\sum\limits_d \bar{N}(y,i,d)\,\bar{w}(i,d)\mathrm{SUIT}(i,d,j,b)}\right)}{\left(\sum\limits_i\sum\limits_d \dfrac{\bar{N}(y,i,d)\,\bar{w}(i,d)\mathrm{SUIT}(i,d,j,b)}{\bar{N}(y,i,d)}\right)\left(\dfrac{1}{\sum\limits_e\sum\limits_h \bar{N}(y,e,h)\,\bar{w}(e,h)\mathrm{SUIT}(e,h,j,b)}\right)} =$$

$$\frac{\dfrac{\left(\dfrac{\bar{N}(y,s,a)\,\bar{w}(s,a)\,\mathrm{SUIT}(s,a,j,b)}{\sum\limits_i\sum\limits_d \bar{N}(y,i,d)\,\bar{w}(i,d)\,\mathrm{SUIT}(i,d,j,b)}\right)}{\bar{N}(y,s,a)\,\bar{w}(s,a)}}{\sum\limits_i\sum\limits_d \dfrac{\left(\dfrac{\bar{N}(y,i,d)\,\bar{w}(i,d)\mathrm{SUIT}(i,d,j,b)}{\sum\limits_e\sum\limits_h \bar{N}(y,e,h)\bar{w}(e,h)\mathrm{SUIT}(e,h,j,b)}\right)}{\bar{N}(y,i,d)\,\bar{w}(i,d)}} =$$

$$\frac{\dfrac{\mathrm{STOC}(s,a,j,b)}{\bar{N}(y,s,a)\,\bar{w}(s,a)}}{\sum\limits_i\sum\limits_d \dfrac{\mathrm{STOC}(i,d,j,b)}{\bar{N}(y,i,d)\,\bar{w}(i,d)}}$$

where the last expression follows from the definition of STOC(s,a,j,b).

APPENDIX G.

The mathematical expression for a selection curve.

As a mathematical model of gear selection we are looking for a sigmoid shaped curve. The curve should e.g. reflect the probability that a fish entering a trawl is retained by the meshes as a function of fish length. Figure Gl shows such a curve

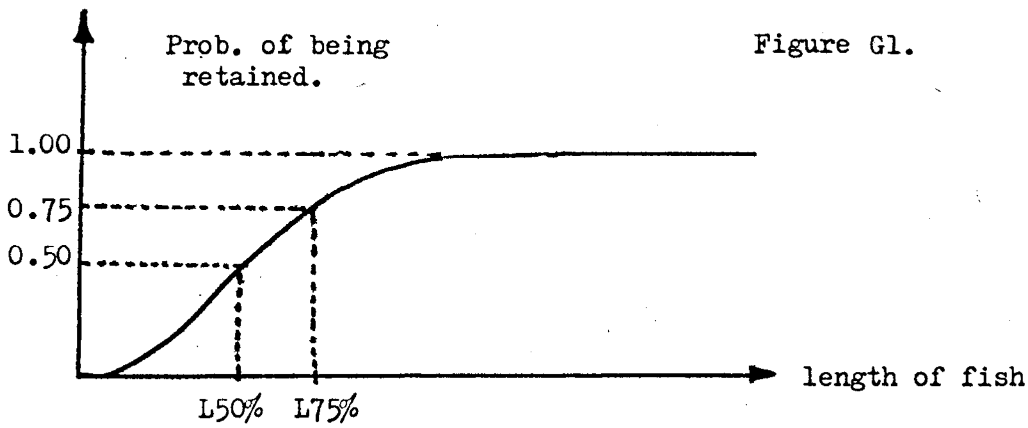

Figure Gl.

L50% is the length of fish at which 50 % of the fish entering the gear are retained and L75% is the length at which 75 % of the fish are retained. L50% and L75% are species and gear specific parameters.
Tanh(L) is a standard mathematical function with a sigmoid shaped graph (see Figure G2).

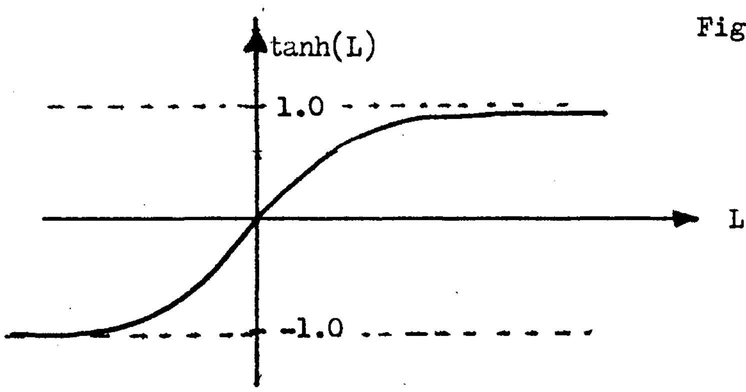

Figure G2.

To "move" the tanh-curve to the appropriate place in the coordinate system and to get the right scale tanh should be multiplied by 0.5 and 0.5 should be added and L50% should be subtracted from the independent variable. The resulting expression becomes

$$0.5 + 0.5 \tanh(L - L50\%) \qquad \text{(G1)}$$

The graph of function (Gl) is given on figure G3.

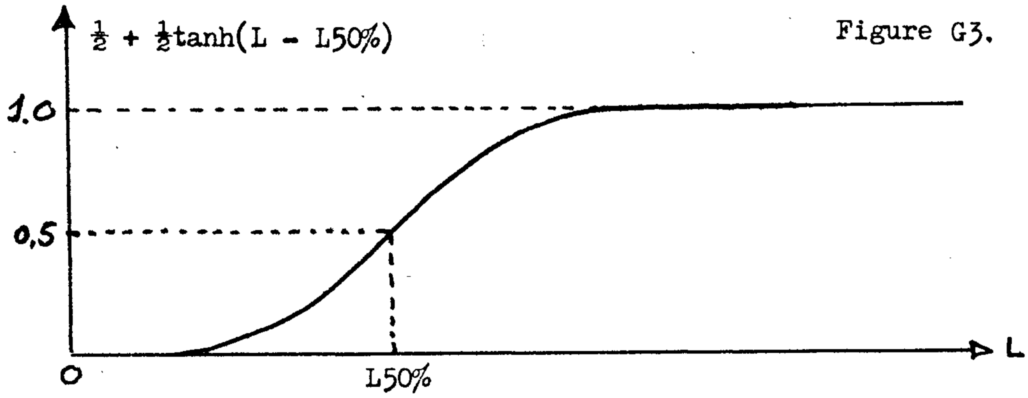

Figure G3.

To obtain a variable steepness of the curve a new parameter alfa is introduced and the function then becomes

$$0.5 + 0.5 \tanh(\text{alfa}(L - L50\%)) \qquad \text{(G2)}$$

where alfa should be given a value so that $0.5+0.5\tanh(\text{alfa}(L75\%-L50\%)) = 0.75$

Inserting the definition of tanh $(\tanh(x) = (\exp(x) - \exp(-x))/(\exp(x) + \exp(-x)))$. we get that $\frac{1}{2}+\frac{1}{2}\tanh(L) = \exp(2L)/(1 + \exp(2L))$ from which we get

$$\frac{\exp(2\text{alfa}(L75\% - L50\%))}{1 + \exp(2\text{alfa}(L75\% - L50\%))} = .75 \qquad \text{(G3)}$$

solving this equatinn with respect to alfa we get

$$\text{alfa} = \ln(3)/(L75\% - L50\%)$$

Writing (G2) as (G3) and inserting the expression for alfa we get

$$\frac{\exp\left(\frac{L - L50\%}{L75\% - L50\%} \ln(3)\right)}{1 + \exp\left(\frac{L - L50\%}{L75\% - L50\%} \ln(3)\right)} \qquad \text{(G4)}$$

The last function (G4) has a graph of tha shape we need.

Other mathematical expressions could have been used, and the reason why this particular formula is chosen is simply that exp is a standard function on all computers.

Appendix H.

A comment on the MSY-concept as defined by the ACFM.

In Report of the Ad. Hoc. Meeting on the Provision of Advice on Biological Basis for Fishery Management (ICES C.M. 1976/Gen:3) the concepts of conditional sustainable yield per recruit and maximum sustainable yield per recruit (MSY/R) were defined.

The MSY/R could be considered as ACFM's proposal for a goal function of fisheries.

In the following it will be demonstrated that the goal function defined in the present work is a generalisation of that defined by the ACFM. Thus the goal function suggested here does not contrast with that defined by the ACFM.

If a number of assumptions are made about the various terms and factors of the goal function suggested by me, we end up with the same results as the ACFM does. The relevant question is whether these assumptions are desirable which I do not think they are.

The assumptions that makes the goal function suggested in this paper equal to MSY/R as defined by the ACFM are:

1) Each stock is in a steady state situation (i.e., constant age distribution of population and catch, constant recruitment and constant mortalities from year to year).

2) Natural mortality is independent of abundance of predators. (i.e., it is ignored that fish eat fish).

3) The fishery on one stock can be managed independently of the management of other fisheries (e.g. it is assumed that the the North Sea fishery on whiting can be managed independently of the North Sea cod fishery).

4) Yields from the various stocks and agegroups landed by the various fleets are assigned the same return-value per kilo (e.g. one kilo of sole is taken as just as good as one kilo of sand-eels).

In the following I attempt to give a formal description of the goal function of the ACFM. Even if the ACFM did not speak about a "goal function", there must be some kind of tacit goal function behind the advice they gave.The Beverton and Holt Y/R formula is based on the assumption of "knife-edge" selection. A more general concept is the Y/R-curve for which no assumption on fishing pattern is made.

Let

$$\underline{P} = (P(0), P(1), \ldots, P(OAGE))$$

be the relative fishing pattern of the stock considered i.e. P(a) is the relative fishing mortality of agegroup a. P(a) is assumed to remain constant during the year. Usually the P's are chosen so that all $P \leq 1$ and P=1 for at least one age group.
(OAGE= the oldest agegroup).
Absolute fishing mortality is defined

$$\underline{F} = (F(0), F(1), \ldots, F(OAGE)) = X \cdot \underline{P} = (XP(0), XP(1), \ldots, XP(OAGE)) \qquad \text{(H1)}$$

Usually $\underline{P}$ is considered constant, (e.g. given by a gear selection curve) and X is usually considered variable. Y/R is usually considered a function of the decision variable X.

If yield per recruit is maximized with respect to X (for a given P) we get the <u>conditional sustainable yield per recruit</u> as defined in <u>Anon.</u> 1976. If yield per recruit is maximized with respect to both X and $\underline{P}$ we get the concept of <u>maximum sustainable yeild per recruit</u> as defined by the ACFM (<u>Anon.</u> 1976).
Let N(o) be the constant number of recruits, and let M(a) be the natural mortality of agegroup a.
Then N(a), the number of survivors in agegroup a (in the beginning of their a'th year of life) is

$$N(a) = N(0)\ \exp\left(-\sum_{i=0}^{a-1} (F(i) + M(i))\right)$$

in the constant parameter model
The yield from agegroup a (during their a'the year of life) is

$$F(a)\ N(a)\ \bar{w}(a)\ (1 - \exp(-Z(a)))/Z(a)$$

where $Z(a)=F(a)+M(a)$ and $\bar{w}(a)$ is the average body weight of age-group a.

Total yields from a yearclass during its life becomes

$$\sum_a F(a)\ N(a)\ \bar{w}(a)\ (1 - \exp(-Z(a)))/Z(a) =$$

$$N(0) \sum_a F(a)\ \exp\left(-\sum_{i=0}^{a-1} F(i)+M(i)\right) \bar{w}(a)\ (1 - \exp(-Z(a)))/Z(a) =$$

$$N(o) \sum_a X\ P(a)\ \exp\left(-\sum_{i=0}^{a-1} XP(i)+M(i)\right) \bar{w}(a)\ (1 - \exp(-XP(a)-M(a)))/(XP(a)+M(a))$$

and yield per recruit as defined by the ACFM (Anon. 1976) is

$$YR(X,\underline{P}) =$$

$$\sum_a XP(a)\ \exp\left(-\sum_{i=0}^{a-1} XP(i)+M(i)\right) \bar{w}(a)\ (1 - \exp(-XP(a)-M(a)))/(XP(a)+M(a)) \quad (H2)$$

in the constant parameter model.

The objective of the ACFM appears to be to maximize

$$YR(X,\underline{P})$$

for each of the stocks assessed by ICES.

In Anon., 1976 the ACFM did not suggest an aggregated goal function accounting for several stocks and several fleets.

The extension of the Y/R-concept to a multispecies concept is problematic. If for example the aggregated goal function is defined as the sum of Y/R from the stocks considered, it becomes :

$$\sum_s \sum_a X(s)\ P(s,a)\ \exp\left(-\sum_{i=0}^{a-1} X(s)P(s,i)+M(s,i)\right) \bar{w}(s,a) \frac{1-\exp(-X(s)P(s,a)-M(s,a))}{X(s)P(s,a)+M(s,a)}$$

$$= \sum_s YR(s,X(s),\underline{P}(s)) \quad (H3)$$

where s is index of species (or stock).

I am not able to give a reasonable interpretation of (H3), due to the fact that the terms of the sum are given in different units.

E.g., is it reasonable to add the yield per sole recruit to the yield per sandeel recruit ? The obvious solution to the problem is to give up the "per recruit" concept and express the terms in more appropriate units (e.g. in units of biomass), but for the moment we shall forget about the inadequateness of (H3).

When the stocks are considered independent the terms of $\sum_s YR(s,X(s),\underline{P}(s))$ can be maximized separately.

If we give up the assumption of independence of stocks the maximization of each stock's Y/R becomes an absurdity. To manage an integrated system towards more than one goal has no meaning. But if we consider $\sum_s YR(s,X(s),\underline{P}(s))$ as the goal, the Y/R concept of the ACFM remains consistent. In that case the goal function might be

$$\sum_s \sum_a X(s)P(s,a) \exp\left(-\sum_{i=0}^{a-1} X(s)P(s,i)+M1(s,i)+M2(s,i)\right) \overline{w}(s,a)$$

$$(1 - \exp(-X(s)P(s,a)-M1(s,a)-M2(s,a)))/(X(s)P(s,a)-M2(s,a)-M1(s,a)) \qquad \text{(H4)}$$

where M2 is the predation induced mortality and M1 is the residual natural mortality (for the definition of M2 see appendix B).

Thus the step from the traditional Y/R to a simple multispecies Y/R does not need to be great.

For the sake of notational convenience let $F(s,a) = X(s)P(s,a)$ and $Z(s,a) = F(s,a)+M1(s,a) +M2(s,a)$. Then (H4) can be written in the short form

$$\sum_s \sum_a F(s,a) \exp\left(-\sum_{i=0}^{a-1} Z(s,i)\right) \overline{w}(s,a)\ (1 - \exp(-Z(s,a)))/Z(s,a) \qquad \text{(H5)}$$

If we give up the yield per recruit concept and replace it by absolute yield the inadequateness caused by the different units of the terms in (H3), (H4) and (H5) is avoided. The unit of the yield equation is biomass per year.

Total yield per year $= \sum_s Y(s,\underline{F}(s)) =$

$$\sum_s \sum_a F(s,a)\ N(s,0) \exp\left(-\sum_{i=0}^{a-1} Z(s,i)\right) \overline{w}(s,a)(1 - \exp(-Z(s,a)))/Z(s,a) \qquad \text{(H6)}$$

The goal function (H6) is based on the assumption of stable stocks and constant recruitment. These assumptions are not fulfilled for any stock covered by an ICES assessment. All fish stocks must be considered as being in a transient state between two steady states and with an extremely low propability of reaching the new steady state within a finite number of years.
If we give up the assumption that the history of one yearclass during its life span equals the history of the entire stock during one year, we obtain a model much closer to our opinion of what actually goes on in the sea. This is easily done (at least from a theoretical point of wiew) simply by putting an extra index on formula (H6)

$$\sum_y \sum_s \sum_a F(y,s,a)\ \exp\left(-\sum_{i=0}^{a-1} Z(y-a+i,s,i)\right) N(y-a,s,a)(1 - \exp(-Z(y,s,a)))/Z(y,s,a)$$

$$= \sum_y \sum_s Y(y,s,\underline{F}(y,s)) \tag{H7}$$

Formula (H7) expresses the yield from a number of yearclasses of a number of species during a period of several years.
By defining the average number of survivors in year y from yearclass y-a by

$$\overline{N}(y,s,a) = \exp\left(-\sum_{i=0}^{a-1} Z(y-a+i,s,i)\right) N(y-,a,s,a)(1 - \exp(-Z(y,s,a)))/Z(y,s,a)$$

(H7) may be written in the short form

$$\sum_y \sum_s Y(y,s,\underline{F}(y,s)) = \sum_y \sum_s \sum_a F(y,s,a)\overline{N}(y,s,a)\ \overline{W}(s,a) \tag{H8}$$

As demonstrated above formula (H8) follows from formula (H2) (the goal function defined by the ACFM) by canceling a number of more or less realistic assumptions.
As formula (H8) is an operational tool for a working procedure of practical assessment, I find it difficult to see why (H2) should be maintained as the goal function of fisheries. Formula (H8) (with or without species interaction) is the straightforward formula we ought to apply until it has been demonstrated that the assumptions behind (H2) are realistic assumptions. However, (H8) is still not satisfactory. The further development of (H8) by the introduction of the "return-value" - concept and by taking into account that most fisheries can not be managed independently of each other is decribed in section 5 of this paper.